Mixed-Convection Heat Transfer in Vertical Packed Channels

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The experimental results of mixed-convection heat transfer in a vertical packed channel with asymmetric heating of opposing walls are reported. The experiments were carried out in the range of 2 < Pe < 2200 and 700 < Ra < 1500. The measured temperature distribution indicates the existence of a secondary convective cell inside the vertical packed channel in the mixed-convection regime. A correlation equation for Nusselt number in terms of Peclet number Pe and Rayleigh number Ra was obtained from experimental data. A plot of Nu/Pe²/³ vs Ra/Pe exhibits the transition of heat-transfer results from the natural convection limit to the forced convection limit. The following three convection regimes exist: natural convection regime, 105 < Ra/Pe; mixed-convection regime, 1 < Ra/Pe < 105; and forced convection regime, Ra/Pe < 1.

Nomenclature

\[ A_{ch} = \text{cross-sectional area of the channel} \]
\[ A_b = \text{surface area of the heated wall} \]
\[ B = \text{shape factor defined in Eq. (5)} \]
\[ c_p = \text{specific heat of Freon-113 (R113)} \]
\[ d_p = \text{sphere diameter} \]
\[ H = \text{plate separation of the channel} \]
\[ K = \text{permeability of the packed bed defined in Eq. (7)} \]
\[ k_f = \text{thermal conductivity of Freon-113} \]
\[ k_m = \text{media effective thermal conductivity} \]
\[ k_s = \text{thermal conductivity of chrome steel} \]
\[ Nu = \text{Nusselt number defined in Eq. (4)} \]
\[ Pe = \text{Peclet number defined in Eq. (3)} \]
\[ q_m = \text{heat flux at the heated wall} \]
\[ Ra = \text{Rayleigh number in Eq. (2)} \]
\[ T_c = \text{temperature of the cooled wall} \]
\[ T_h = \text{temperature of the heated wall} \]
\[ T_i = \text{fluid temperature at inlet of the test section} \]
\[ u_i = \text{fluid velocity at inlet of the test section} \]
\[ \sigma_m = \text{media thermal diffusivity, } \alpha_m = k_m/(\rho c_p) \]
\[ \beta = \text{thermal expansion coefficient} \]
\[ \Delta T = \text{temperature difference, } \Delta T = T_h - T_c \]
\[ \lambda = \text{thermal conductivity ratio of } k_f \text{ and } k_s \]
\[ \nu = \text{kinematic viscosity of Freon-113} \]
\[ \rho = \text{density of Freon-113} \]
\[ \psi = \text{porosity of the packed bed} \]

Introduction

CONVECTIVE heat transfer in a porous media packed channel has been a subject of intensive study during the past two decades because its wide applications including geothermal energy engineering, groundwater pollution transport, nuclear waste disposal, chemical reactor engineering, insulation of buildings and pipes, and storage of grain and coal, and so on. Most of the previous work has been devoted to the studies of either natural or forced convection in porous media. There are only a few papers on numerical and theoretical studies of mixed convection in a packed channel. For example, Wooding, Prats, and Sutton conducted theoretical studies on the onset of free convection in porous channels. Haaji and Tien presented their analytical and numerical results on mixed convection in a horizontal porous channel. For uniformly heated horizontal porous channel Islam and Nandakumar numerically investigated the problem of buoyancy-induced secondary flow in a porous duct with a rectangular cross section at large Peclet numbers. The discovery was made that low Rayleigh numbers a secondary flow pattern of two counter-rotating cells exist. With the increase of the Grashof number, the strength of the secondary flow is increased, and a strong boundary layer behavior is exhibited on the vertical side wall; thus, the heat-transfer rate is significantly enhanced. When the Grashof number is increased, a stable four-vortex pattern takes place, and the heat-transfer rate increases accordingly. Prasad et al. conducted a numerical study of mixed convection in a horizontal porous layer with discrete heat sources from below. At small Peclet numbers a thermal plume rises above the heat source, and a pair of counter-rotating cells is generated. With an increase in the Peclet number, the symmetric nature of the isotherm vanishes; the strength of the two recirculating cells becomes weaker, and the thermal plume moves downstream. Muralidhar numerically studied both horizontal and vertical annuli with the inner cylinder heated and the outer cylinder cooled. He found that mixed convection is predominant for 0 < Pe < 10 and Rayleigh number 0 < Ra < 500. Kwendakwema and Boehm obtained a numerical solution for mixed convection about a vertical concentric cylinder in the range of 0.1 < Gr Da/Re < 10. They pointed out that the radius ratio three is a critical value above which the increase in heat transfer is minimal. Lai numerically investigated aiding and opposing mixed convective flows in a vertical porous layer for the case when a finite isothermal heat source is located on a vertical wall while the other wall is isothermally cooled. They found that a circulatory secondary flow exists. For an aiding flow the heat-transfer rate increases monotonically with the aiding velocity. For the opposing flow with increasing Peclet numbers, the heat-transfer rate first decreases and reaches a minimum before starting to increase again. For a vertical porous annulus with the same thermal boundary condition, Choi et al. confirmed the results of Lai and suggested that heat-transfer results Nu/Pe²/³ should be correlated in terms of Ra/Pe for an isothermal heat source, whereas Nu/Pe²/³ should be correlated in terms of Ra/Pe² for the case of constant heat flux. Chou et al. conducted a numerical investigation on fully developed non-Darcian mixed convection in a horizontal porous channel by assuming a no-slip boundary flow, with the effects of inertia, channeling, and thermal dispersion taken into account. 

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into consideration. They found that there exists a higher secondary flow velocity, especially in the region near the vertical wall caused by the effects of buoyancy and channeling. The numerical results also show that both the secondary flow pattern and the heat-transfer rate are significantly affected by the buoyancy force when the Peclet number is low. They found that the buoyancy effect is suppressed by the Peclet number dispersion effect when the Peclet number is increased.

Early experimental work on mixed convection is given by Elder,12 Combarnous,13 and Schrock and Laird.14 Recently, Renken and Poulidakos15 performed an experiment on mixed-convection heat transfer in a horizontal packed-sphere channel heated from below. Their experimental results show the dependency of the temperature field in a porous bed, the growth of the thermal boundary layer, and the variation of local Nusselt number on the parameter \( \frac{Ra}{Pe^3/2} \). Their experimental results lie in the forced convection limit of the mixed convection regime. Reda16 performed both experimental and numerical investigation on mixed-convection heat transfer in a vertical annulus packed with glass spheres. His numerical results show that in the presence of superimposed downstream the buoyancy-induced upflow is first retarded, then stagnated, and ultimately suppressed with increasing magnitude of the downstream Peclet number. Although radial temperature profiles were given in this paper, no heat-transfer results were presented. Clarke and Kulacki17 conducted an experiment on mixed-convection heat transfer between vertical concentric cylinders filled with a porous medium. Temperature distributions and the local Nusselt number in a porous bed were measured. However, the experiments were performed at the forced convection limit of mixed-convection regime, and there are no data for heat-transfer coefficient in the transitional regime from the natural convection to the forced convection limit. Choi and Kulacki18 presented their numerical and experimental results on mixed convection through vertical porous annuli, which is heated from the inner cylinder with constant heat flux. They found that the Nusselt number increases with either \( \frac{Ra}{Pe^3/2} \) or \( \frac{Pr}{D^8} \) for the aiding flow, whereas for the opposing flow it decreases with increasing magnitude of the Peclet number. They also found that the mixed convection regime for the aiding flow is \( 0.2 < \frac{Ra}{Pe^3/2} < 9800 \), whereas for the opposing flow it is \( 0.6 < \frac{Ra}{Pe^3/2} < 17,100 \). The preceding review of the literature reveals that all of the experimental studies on mixed convection in porous media are confined to either horizontal porous layer or vertical annuli.

In this paper an experiment was carried out to study mixed convection of R-113 (\( \Pr = 8.06 \)) in a vertical channel (with \( L/H = 9 \)) packed with chrome steel spheres and heated asymmetrically. Transverse temperatures at five elevations in the test section and the heat flux to wall were measured. The strength and extent of the convective cell strongly depend on the parameters of \( Ra \) and \( Pe \). A correlation equation of Nusselt number in terms of \( Pe \) and \( Ra \) is obtained. The transition regime from natural to forced convection limits is also presented.

**Experimental Method**

A forced convection of R-113 loop was used in performing the experiments. Details of the loop are described elsewhere.19 A schematic of the test section is illustrated in Fig. 1. The test section is vertically oriented with R-113 flowing against gravity. It consisted of four vertical walls: two aluminum plates serving as the heat source and the heat sink and two acrylic plates serving as adiabatic side walls. Chrome steel beads (\( d_p = 6.35 \text{ mm} \)) were used as porous media. The overall dimensions of the test section were 66.04-cm length, 20.32-cm width, and 30.48-cm depth. The cross-sectional flow area of the test section was 5.08 × 15.24 cm. The heated and cooler plates, 45.72 cm in length (\( L \)) and facing opposite from each other, were separated by a distance (\( H \)) of 5.08 cm. In this work the porosity was determined by measuring the volume ratio of filled water in the packed channel to the empty channel. The heat-transfer section is preceded by a calming section of 10.16 cm in length and followed by an exit section of 5.08 cm in length. Both the calming and the exit sections were made of acrylic plates. A perforated plate was installed on the bottom of the test section to promote a more uniform velocity profile entering the calming section while a top perforated plate was used to hold the spheres in place during the experiments. Both the walls and the perforated plates were removable, so that the spheres can be packed easily in the test section. An aluminum plate, 20.32 × 45.72 cm and 3.175 cm thick, was used as the heated plate. The back of the aluminum plate was mounted with five strip heaters, each having a maximum heating capacity of 350 W. The energy output of each strip heater was controlled by a rheostat. The entire heated test section was insulated with fiberglass. The accuracy of the power input to the heated plate was estimated to be ±3%. The cooling jacket was made from a 7.62-cm-thick aluminum plate. A 6.82-cm-deep zigzag track, which was machined into the rear side of the cooled plate, provided the cooling water passage. Twelve J-type thermocouple probes (1.59 mm in diameter) in each plate were positioned at a distance of 1.59 mm from the inner walls to monitor the temperatures of the heated and cooled plates. The other two side walls were made of acrylic plates. Five levels of J-type thermocouple probes (0.81 mm in diameter), located at 6.35, 9.21, 18.1, 27.0, and 45.1 cm from the inlet, were mounted on these two side plates of the test section. Each level has 11 thermocouples in the transverse direction with an even spacing of 4.76 mm except those near walls. The thermocouples near the walls were located 1.59 mm from the walls. The thermocouples, installed in parallel with the heated and cooled plates, were used to measure the transverse fluid temperature in the packed channel. The heat loss through these two side plates was estimated to be less than 1% of the power input to the heated plate.

A data acquisition system consisting of a personal computer, an A/D converter board, six universal analog input multiplexers, and a screw terminal accessory board was employed to record and display the flow rate of R-113 and the measured temperatures. The accuracy of the flow rate measurement was ±0.5%. The experimental accuracy of the temperature was ±0.2°C. Uncertainties of the Nusselt number were estimated to be ±4.8%.

**Similarity Parameters**

We now consider the test section as just described and illustrated in Fig. 1. The height of the vertical channel is \( L \), and the separation
$H$. Both of the two vertical walls are maintained at uniform temperature while one is heated at $T_h$ and the other is cooled at a constant value $T_c$. The working fluid enters the packed channel at a uniform velocity $u$. To obtain a fully developed flow in a short channel heated asymmetrically, the fluid temperature at the inlet of the test section is maintained at a temperature $T_i$, which is of the average value of $T_h$ and $T_c$, i.e., $T_i = (T_h + T_c)/2$. For the present problem the heat-transfer rate between the hot wall and the cold wall of the packed channel in the range of mixed convection is correlated in terms of the Rayleigh number and the Peclet number$^{20}$ i.e.,

$$Nu = f(Ra, Pe)$$  \hspace{1cm} (1)

where $Ra$, $Pe$, and $Nu$ are defined as

$$Ra = \frac{\gamma \beta K \Delta T H}{\nu \alpha_m}$$  \hspace{1cm} (2)

$$Pe = u H / \alpha_m$$  \hspace{1cm} (3)

$$Nu = q_m H / \kappa_m \Delta T$$  \hspace{1cm} (4)

respectively, $g$ being the gravity. The effective thermal conductivity $\kappa_m$ is calculated by using Zehner and Schlunder’s equation$^{21}$:

$$\kappa_m = k_s \left\{ 1 - \sqrt{1 - \varphi + 2 \frac{1 - \varphi}{\lambda B} \left[ \lambda B^{-1} \frac{1}{(1 - \lambda B)^2} \varphi \right]} \right\}$$  \hspace{1cm} (5)

with $k_s$ and $k_f$ being the thermal conductivity of solid and liquid phases, respectively. A shape factor $B$ for a packed bed consisting of uniform sphere is given by

$$B = 1.25 [(1 - \varphi) / \varphi]^{1/3}$$  \hspace{1cm} (6)

The permeability $K$ in Eq. (2) is given by

$$K = \frac{\varphi^3 d_p^2}{180 (1 - \varphi)^2}$$  \hspace{1cm} (7)

**Results and Discussion**

The experiments were conducted under a wide range of conditions from the natural convection limit to mixed convection and to the forced convection limit. The Peclet number and Rayleigh number were $2 < Pe < 2200$ and $700 < Ra < 1500$, respectively. In what follows, we shall first present some typical transverse temperature distributions in the test section. The transition regimes from natural to forced convection limits will then be discussed. Finally, a correlation equation of $Nu$ in terms of $Pe$ and $Ra$ will be presented.

**Temperature Distribution**

Temperature distributions at the five levels for three different values of $Ra/Pe$ are presented in Fig. 2. Figure 2a shows the temperature distribution for the forced convection predominated case with $Ra/Pe = 0.32$. When forced convection predominates, buoyancy force is unimportant; thus, temperature distributions at different elevations are asymmetric with respect to the centerline. The temperature field can be divided into three regions: two thermal boundary layers near the heated and cooled walls and an isothermal core at the inlet temperature. The temperature near the heated wall increases as it approaches downstream because of the heating effect from the wall. On the other hand, the temperature near the cooled wall decreases as it approaches downstream because of the cooling effect from the wall. Figure 2b illustrates the temperature distributions for the mixed convection case with $Ra/Pe = 11.3$. The shape of the temperature distribution at $x = 6.35$ cm is very similar to the case of forced convection, which is asymmetric with respect to the centerline. As the fluid approaches downstream, buoyancy effect becomes increasingly important, and temperature distribution begins to deviate from those of asymmetric shape. Note that the temperature at $x = 45.1$ cm near the cooled wall is higher than that at $x = 6.35$ cm, suggesting the existence of a buoyancy-driven secondary cell. Lai et al.$^9$ also reported the existence of the secondary convective cell in the mixed convection regime in their numerical results for streamlines. Figure 2c shows the temperature distribution for the natural convection predominant case with $Ra/Pe = 300$. Note that the thermal boundary layer becomes very thick downstream, which is almost constant across the channel with a large temperature drop near the cooled wall. Because the temperature of the fluid at $x = 45.1$ cm is much higher than the inlet temperature, this indicates the existence of a secondary cell.

To investigate the existence of the buoyancy-driven secondary cell, temperature distributions for $Ra = 700$ along three different levels at different Peclet numbers are plotted in Fig. 3. We shall focus our attention to the temperature distributions near the cooled wall where a secondary cell exists at low Peclet numbers. As shown in Fig. 3a, the temperature near the cooled wall at $x = 6.35$ cm (i.e., at the first level) decreases monotonically as the Peclet number is decreased because at the entrance the buoyancy effect caused by the heated or cooled wall is unimportant, as mentioned earlier. Temperature distributions at $x = 18.1$ cm are presented in Fig. 3b. As the Peclet number is decreased from 2200 to 125, the temperature near the cooled wall begins to drop. However, a further decrease in
The Peclet number leads to an increase in fluid temperature near the cooled wall because of the existence of a secondary cell that brings hotter fluid from the top. This effect is most pronounced in Fig. 3c, where the fluid temperatures for $Pe = 2$ and $50$ at $x = 45.1$ cm is higher than that for $Pe > 50$ at $Ra = 700$, which confirms the existence of a convective cell.

The effects of the Rayleigh number on temperature distributions at $x = 6.35$ and $45.1$ cm in the mixed convection regime with $Ra/Pe = 28 \sim 60$ are presented in Figs. 4a and 4b, respectively. As is expected, the fluid temperature for an aiding flow at a given Peclet number increases as the Rayleigh number is increased.

**Nusselt Numbers**

Variations of the Nusselt number with the increase of the Peclet number at selected Rayleigh numbers are presented in Fig. 5. At lower Peclet numbers the effect of the Rayleigh number is significant. In this regime the higher the Rayleigh number, the higher is the Nusselt number. When the Peclet number is larger than a certain value, the heat flux data points merge for different Rayleigh numbers, which means that the Nusselt number becomes independent of Rayleigh number and forced convection prevails. This observation can also be confirmed by Fig. 6, where the variation of the Nusselt number is plotted against the Rayleigh number at selected Peclet numbers. As shown in Fig. 6, at lower Peclet numbers such as $Pe = 2$, 10, and 125 the Nusselt number increases with increasing the Rayleigh number. This suggests that the natural convection is predominated at lower Peclet numbers. On the other hand, however, when the Peclet number is larger than a certain value ($Pe = 500$), the variation of the Nusselt number becomes independent of the Rayleigh number, indicating that the forced convection becomes predominant.

The following correlation was obtained based on a least-square fit of 81 experimental runs for mixed convection of R-113 in a vertical packed channel heated asymmetrically with uniform temperature:
An experimental investigation on mixed-convection heat transfer in a vertical packed channel with asymmetric heating thermal boundary conditions has been conducted. The measured temperature distributions show that there exists a secondary convective cell in the mixed-convection regime. A correlation equation of $Nu$ in terms of $Ra/Pe$ has been obtained based on 81 experimental runs in the range of $2 < Pe < 2200$ and $700 < Ra < 1500$. The correlation equation agrees well with the experimental data. The mixed-convection regime for a vertical packed channel is $1 < Ra/Pe < 105$.

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**References**