Helical domain patterns in tube configurations: effect of geometry length scales

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Key words: martensitic phase transformation, superelastic NiTi tubes, self-organized helical domain patterns, tube geometry, bending strain energy and domain front energy.

Abstract. Superelastic NiTi polycrystalline tubes, when subjected to quasi-static stretching, transform from an initial austenite phase to a high-strain martensite phase by the formation and growth of a macroscopic self-organized helical domain as deformation progresses. This paper performed an experimental study on the effects of the externally applied stretching and tube geometry (length $L$, wall-thickness $h$ and tube radius $R$) on the martensitic helical domains in the tubes under very slow (isothermal) stretching. The evolution of the helical domains with the applied strain in different tube geometries are quantified by in-situ optical measurement. We demonstrate that the shape of the self-organized helical domain and its evolution are governed by the competition between bending strain energy and domain front energy in minimizing the total energy of the tube system. The former favors a long slim helical domain, while the latter favors a short fat helical domain. The experimental results provide a strong support to the recently developed theoretical relationship.

Introduction

In recent experiments [1-3], phase transition in superelastic NiTi polycrystalline tubes under displacement-controlled quasi-static uniaxial stretching is localized in a helix-shaped martensite (M) domain that consists of almost fully transformed grains with constant average strains (elastic strain plus transformation strain) of about $\varepsilon_{\text{xy}}=6\%$, $\varepsilon_{\text{zz}}=3\%$ and $\varepsilon_{\text{xx}}=3\%$ in axial (y), radial (z) and circumferential (x) direction respectively (Fig. 1 (a)). Such self-organized domain is separated from the homogeneous elastically deformed austenite (A) matrix (with an axial strain of $\varepsilon_{\text{xy}}=1\%$) by the domain front of a finite thickness. Under continued stretching, this high-strain helical domain grows via the propagation of the domain front (a macroscopic interface) before it is eventually merged into a cylindrical domain [3-5]. Generally speaking, helical domain patterns and their evolutions are governed by competition of different energy terms under the constraint of the tube geometry. For a tube containing one helical domain, two energy terms are distinct and must be emphasized. The first is the bending strain energy caused by the overall elastic misfit in tube circumference between the untransformed austenite and the partially transformed region (Fig. 1 (a)), and is sensitive to the domain pattern (long slim or short fat). The second is the front energy of the helical domain, arising from the strain misfit along tube wall-thickness direction between A and M phases. There is almost no in-plane mismatch across the domain front in the tube mid-surface since the front always orients at about 55° to the tube axis (i.e., an invariant-line as shown by LL’ in Fig. 1 (b)) [6, 7]. This fact makes the domain front energy density scales with the thickness of the tube-wall [8]. More important, competition between the domain front energy and the bending strain energy in minimizing total free energy of the tube system plays an important role in the observed helical domain patterns.

In this paper, we aim to quantify the above two distinct energy terms by direct in-situ measurement of the morphology of the helical domain (such as number of the helical coils, $N$). The variation of $N$ under different applied stretching (nominal transformation strain $\varepsilon_{\text{tr}}$) and in different tube geometries ($L$, $h$, $R$) will be quantified. The experimental results are compared with the recently developed theoretical modelling [9].
Fig. 1. (a) Scheme of the helical domain in a tube and the overall necking leading to the bending energy of the tube; (b) the macroscopic misfit between A and M domains along the wall-thickness direction.

Materials and experimental setup

The specimens used in the experiment are commercial polycrystalline NiTi tubes with grain sizes of around 80nm (NDC company, USA). A total of 7 specimens were prepared from three different tubes (T1, T2 and T3; as listed in Fig. 5). They were all mechanically polished by fine grained sand papers to reach a final surface roughness of about 0.15μm. The austenite finish temperatures ($A_f$) for all the tubes are below the testing temperature (23 °C), so the material is in the austenite state and will exhibit superelastic behavior under stress. The tubes were all tested in a standard mechanical testing machine (Model UTM-RT/10, MTS) at a very slow nominal strain rate of $1.0 \times 10^{-5}$ s$^{-1}$. Each test was conducted on a fresh specimen. The macroscopic helical domain morphologies during the loading were recorded by a CCD camera (Model TK-C1381, JVC) with 25 frames per second (fps). As shown in Fig. 1, the axial length of the helical domain $L_M$ is related to the number of the helical coils $N$ by:

$$N = \frac{L_M}{l_p}.$$  

(1)

The variation of $N$ in the loading process was synchronized with the measured nominal stress-strain curves [3], and all the tube samples were unloaded just before the helical domain is merged into a cylindrical domain. The geometric misfits in radial and wall-thickness directions due to the presence of the helical domain were measured by the optical profiler (Model Wyco NT3300, VEECO) so as to demonstrate the bending strain energy and the domain front energy.

Experimental results and discussion

Bending strain energy due to radial misfit. The presence of a helical domain (with transformation strain $\varepsilon_{yy}^{tr}=5\%$, $\varepsilon_{xx}^{tr}=-2.5\%$, $\varepsilon_{zz}^{tr}=-2.5\%$ in the axial, circumferential and wall-thickness direction) causes a overall reduction in tube circumference and local reduction in tube wall-thickness as schematically shown in Fig. 1 (a), (b) respectively [9]. The reduction in the circumference leads to an overall inward radial necking which misfits with the untransformed section of the tube. The necking
profiles of the tube generator at different stages of domain growth were measured by optical profiler as shown in Fig. 2 (a). The bending strain energy due to the necking misfit is mainly stored in the two transition regions with characteristic length $l_b$ (as shown in Fig. 2 (a)). By geometric analysis, the overall necking depth $w_{tr}$ due to the presence of the helical domain is

$$w_{tr} = R \cdot \varepsilon^{\mu}_{xx} \cdot (l_m/l_p) = R \cdot (\varepsilon^{\mu}_{yy}/2) \cdot SF$$  \hspace{1cm} (2)

where $SF = l_m/l_p$ is the slimness factor indicating the shape of the helix, $l_m = (L \cdot \varepsilon_{x})/(N \cdot \varepsilon^{\mu}_{yy})$ is the axial width of each helical coil. As shown in Fig. 2 (b), the measured value of $w_{tr}$ was found to increase linearly with $SF$ (for the case D-5, $SF=1$) of the helix, agreeing well with Eq. 2. The bending strain energy will be calculated later.

Fig. 2. (a) 5 measured domain patterns in T2-S2 ($R=0.78\text{ mm}$) for the calculation of the tube mid-surface deflection; (b) the linear dependence of the measured necking depth on domain patterns (i.e. slimness factor of the helix).
Domain front energy due to tube wall-thickness misfit. The reduction of tube-wall thickness in the martensite domain causes a misfit between A and M along tube wall-thickness (z) direction. The misfit creates macroscopic misfit stress and is the source of macroscopic domain front energy which is proportional to the interface (front) area. There is a similar case in the plate configuration. We used thin plate specimens (Fig. 3 (a)) under tension to determine the frontal region (front thickness $l_a$), local thickness reduction ($\Delta h$) and the dependence of the domain front energy density on the plate thickness ($h$). From the measured axial front thickness $l_a = l_c \cos \phi$ with $\phi$ the orientation angle ($\phi = 35^\circ$ as shown in Fig. 3 (a)), it was found that both $l_a (\approx h$, so $l_a = h \cos \phi$) and $\Delta h (= 2.54\% h)$ are proportional to the wall thickness $h$, as indicated in Fig. 3. We can estimate domain front energy density $\gamma_{\text{front}}$ as

$$\gamma_{\text{front}} = \lambda \cdot l_c \cdot E \cdot (\varepsilon_{zz}^c)^2$$

(3)

where $\lambda$ is a coefficient which only depends on Poisson’s ratio $\nu$ of the material.

![Fig. 3. (a) Surface profiles of domains in strips of different thicknesses; (b) the dependence of the local thickness misfit ($\Delta h$) and axial front thickness $l_a$ on strip thicknesses $h$.](image-url)
Minor twist strain energy. There is little overall twist of the transformed section with reference to the untransformed section of the tubes despite a relative kink between austenite and martensite, as shown in Fig. 4. The relative kink is measured to be $2^\circ$ which agrees well with the theoretical calculation [9]. Furthermore, the total overall twist $\theta_{\text{total}}$ can be analytically estimated as

$$\theta_{\text{total}} = \frac{l_M}{l_p} \left[ \left(1 + \varepsilon_{yy}^{\text{tr}}\right) \cdot \alpha - \varepsilon_{yy}^{\text{tr}} \cdot \tan \phi \right] = \frac{l_M}{l_p} \cdot 0.00195 \approx \frac{l_M}{l_p} \cdot 0.1^\circ < 0.1^\circ.$$  

(4)

This small twist causes a very small rotation misfit between the transformed and untransformed sections. However, as shown in Fig. 4 (b), the tube generator almost maintains straight even with the presence of the helical domain. This supports the estimation by Eq. 4 and indicates that the strain energy due to the twist can be ignored.

Energy minimization and scaling law of equilibrium helical domain

According to Ref. [9], the bending strain energy $U_{\text{bend}}$ of the tube can be expressed as

$$U_{\text{bend}} = 2\pi R \cdot h \cdot l_b \cdot \frac{E}{2R} \cdot \left(\frac{W_p}{l_b}\right)^2 = A_1 \cdot \frac{1}{N^2} \cdot \frac{h}{R} \cdot \left(L \cdot \varepsilon_\gamma\right)^2.$$  

(5)

where $A_1 = \frac{E}{32\pi \cdot \tan^2 \phi}$, $l_b = \left(\frac{1}{3(1-\nu^2)}\right)^{\frac{1}{3}} \sqrt{R \cdot h}$ is the characteristic length of the necking transition zone. The total domain front energy $\Gamma$ of the helix can be expressed as

$$\Gamma = N \cdot \frac{4\pi \cdot R \cdot h \cdot \gamma_{\text{front}}}{\cos \phi}.$$  

(6)

It is seen that, for a given $\varepsilon_\gamma$ (given amount of martensite), the bending strain energy inversely scales with $N$ by an exponent 2, while domain front energy linearly scales with $N$. From energy point of view, the bending strain energy favors a long (large $N$) helical domain, while the domain front energy favors a short (small $N$) helical domain.

We can apply the energy minimization principle to determine the coil number $N$ of the self-organized helical domain. It is shown that the $N$ depends on the tube geometry ($L$, $h$, $R$) and nominal transformation strain ($\varepsilon_\gamma$) by the following scaling law

$$N = K_{th} \cdot \left(\frac{R}{h}\right)^{\frac{1}{6}} \cdot \left(\frac{L \cdot \varepsilon_\gamma}{R \cdot \varepsilon_{yy}^{\text{tr}}}\right)^{\frac{2}{3}} \cdot \frac{1}{3(1-\nu^2)}^{\frac{1}{3}} \cdot \frac{\cos \phi}{16 \cdot (\pi \cdot \tan \phi)^2},$$  

(7)
The experimental measurements of $N$ in the 7 tube samples used in this work are shown in Fig. 5. It is seen that $N$ depends on the product of two dimensionless quantities $(R/\ell)^{\frac{1}{2}} \cdot (L\ell_{c}/R\varepsilon_{yy})^{\frac{2}{3}}$ in a linear manner, which strongly supports Eq. 7 (as shown in Fig. 5, $K_{th} = 0.95$).

Fig. 5. The measured coil number $N$ of the helical domain in 7 different NiTi tube geometries used and the theoretical prediction by Eq. 7.

Conclusions
In this paper, the coil number of the self-organized helical domains ($N$) under different externally applied stretching (nominal transformation strain $\varepsilon_{xx}$) and in different tube geometries were measured. We have quantified the two important energy terms, bending strain energy and domain front energy, in NiTi tubes under tension. It is shown that both energy terms depend on externally applied stretching ($\varepsilon_{xx}$), tube geometry ($L$, $h$, $R$) and domain morphology. We show that it is the competition between these two energy terms in minimizing total free energy of the tubes that determines the observed helical domain patterns. The experiment on the helical domain shapes provides strong support for the scaling law derived recently from theoretical analysis.

Acknowledgement
The authors are grateful to the financial support from the Research Grants Council of the Hong Kong SAR, China through Project No. 619806.

References
Solid-Solid Phase Transformations in Inorganic Materials
doi:10.4028/www.scientific.net/SSP.172-174

Helical Domain Patterns in Tube Configurations: Effect of Geometry Length Scales
doi:10.4028/www.scientific.net/SSP.172-174.1090