Effect of deformation frequency on temperature and stress oscillations in cyclic phase transition of NiTi shape memory alloy

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Abstract
Distinctive temperature and stress oscillations can be observed in superelastic shape memory alloys (SMAs) when they subject to displacement-controlled cyclic phase transition. In this paper, we examine the effect of the deformation frequency on the thermal and mechanical responses of the polycrystalline superelastic NiTi rods under stress-induced cyclic phase transition. By synchronized measurement of the evolutions in overall temperature and stress–strain curve over the frequency range of 0.0004–1 Hz (corresponding average strain rate range of 4.8 × 10⁻⁵/s—1.2 × 10⁻³/s) in stagnant air, it was found that both the temperature evolution and the stress–strain curve vary significantly with the frequency and the number of cycles. For each frequency, steady-state cyclic thermal and mechanical responses of the specimen were reached after a transient stage, exhibiting stabilization. In the steady-state, the average temperature oscillated around a mean temperature plateau which increased monotonically with the frequency and rose rapidly in the high frequency range due to the rapid accumulation of hysteresis heat. The oscillation was mainly caused by the release and absorption of latent heat and increased with the frequency, eventually reaching a saturation value. The variations in the stress responses followed similar frequency dependence as the temperature. The steady-state stress–strain hysteresis loop area, as a measure of the material’s damping capacity, first increased then decreased with the frequency in a non-monotonic manner. The experimental data were analyzed and discussed based on the simplified lumped heat transfer analysis and the Clausius–Clapeyron relationship, incorporating the inherent thermomechanical coupling in the material’s response. We found that, for given material’s properties and specimen geometries, all such frequency-dependent variations in temperature, stress and damping capacity were essentially determined by the competition between the time scale of the heat release (i.e. the phase transition frequency) and the time scale of the heat transfer to the ambient. The results emphasize that, the two time scales of loading and heat transfer must be clearly specified when characterizing and modeling the cyclic behavior of SMA materials.

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1. Introduction

Superelastic NiTi shape memory alloys (SMAs) are increasingly used in many fields, from biomedical human artificial joints to small-scale sensors and actuators in micro-electro-mechanical-systems (MEMS). This material also has potential applications in devices of vibration and damping control for large and small scale engineering structures. These applications utilize the material’s large deformation and damping capacity from reversible transformations between austenite and martensite phases. Miyazaki et al. (1986) first found that residual strain, transformation stress and the hysteresis loop changed significantly during cyclic deformation. Since the fatigue and degradation of the devices strongly rely on the thermal and mechanical responses of the material in the cyclic transformation (see Stalmans et al., 1994; Strnad et al., 1995; Abeyaratne and Kim, 1997; Tobushi et al., 1997; Otsuka and Wayman, 1998; Hornbogen, 2002; Dolce and Cardone, 2005; Nemat-Nasser and Guo, 2006; Zaki and Moumni, 2007; Demers et al., 2009; Kang et al., 2009; Morin et al., 2011a, 2011b), performances of the material under cyclic loading are always of key concern in the applications (Piedboeuf et al., 1998; Dolce and Cardone, 2001; Lim and McDowell, 2002; Saadat et al., 2002; Puglisi and Truskinovsky 2005; Dolce and Cardone, 2005; Predki et al., 2006; Pan and Cho, 2007; Soul et al., 2010). One important aspect of the mechanical behaviors of SMA material is the strong rate or frequency dependence (Van Humbeeck and Delaey, 1981; Piedboeuf et al., 1998; Tobushi et al., 1998; Gandhi and Wolons, 1999; Liu et al., 1999; Schmidt and Lammering, 2004; Pieczyska et al., 2005; Zhu and Zhang, 2007; Grabe and Bruhns, 2008; Soul et al., 2010; He and Sun, 2010a, 2010b; Zhang et al., 2010; Amini et al., 2011; He and Sun, 2011; Morin et al., 2011a, 2011b; Sun et al., 2012; Yan et al., 2012; Ahadi and Sun, 2013; Yin et al., 2013) due to heat generation, heat accumulation, heat transfer and the intrinsic temperature dependence of the transformation stress of the material (Clausius–Clapeyron relation). Many researchers (see Van Humbeeck and Delaey, 1981; Leo et al., 1993; Ortin, 1995; Shaw and Kyriakides, 1995, 1997; Brinson et al., 2004; Petrýk, 2005; Vitiello et al., 2005; Grabe and Bruhns, 2008; Bernardini and Rega, 2010; He et al., 2010; Morin et al., 2011a, 2011b; among many others) have shown that there exists a complex interaction and coupling among the stress, internal heat sources and temperature fields. The stress–strain responses of the material in the cyclic phase transitions are intrinsically non-isothermal and are accompanied by distinctive oscillations in the strain and the temperature fields (Zhu and Zhang, 2007; Heller et al., 2009; Torra et al., 2010; He and Sun, 2010a; Soul et al., 2010; Morin et al., 2011a, 2011b; Yin, 2013). Both the amplitude of the oscillation and the mean value of the temperature vary significantly with the deformation frequency. From both academic and practical points of view, a comprehensive understanding of the coupling effect and the roles of the time scales of phase transition (or the frequency of the loading) and heat transfer in the thermal and mechanical responses of the material is required (Ortin and Delaey, 2002; He and Sun, 2010a; Sun et al., 2012). However, these important aspects remain relatively unexplored, both
experimentally and theoretically. Systematic, well controlled experiments over a wide range of deformation frequencies play critical roles in obtaining first hand data and in comprehending and modeling such multi-scaled phenomena. At the same time, in order to have a clear picture of the non-isothermal cyclic responses of the NiTi material, the following key physical factors must be taken into consideration:

1. **Internal heat sources**: From the continuum mechanics point of view, a nonlinear, non-monotonic constitutive equation can be used to describe the first order phase transition process in NiTi polycrystals where typical instability events of nucleation of high strain domains, at both micro- and macro-levels, are involved in the process (Shaw and Kyriakides, 1997; Brinson et al., 2004; Heller et al., 2009; Torra et al., 2010; Zhang et al., 2010; Sun and He, 2008; He and Sun, 2009, 2010a, 2010b, 2010c; Soul et al., 2010; Delpueyo et al., 2011, 2012). This localized transformation (deformation) also serves as the heterogeneous heat source since it is accompanied by the release of the latent heat and the hysteresis heat of the material at the macroscopic level. The hysteresis heat released is approximately equal to the area of the stress–strain hysteresis loop in each specific cycle, ignoring the energy transmitted to the ambient surroundings by acoustic waves.

2. **Heat transfer**: The generated heat will transfer via conduction within the specimen (at the two clamping ends as well) and through convection between the specimen and the external ambient, leading to a swift temperature variation of the material and strongly affecting the overall stress–strain responses since the stress is temperature dependent (Clausius–Clapeyron relation).

3. **Competition of different physical processes that have different characteristic time scales**: At the macroscopic level, the heat generation and exchange between the specimen and the environment (via the clamping grips and the ambient) under cyclic phase transition depend on both the deformation frequency (providing a time scale of the heat release) and the ambient condition (defining the time scale of heat transfer to the outside) (Ortin and Delaey, 2002; He and Sun, 2010a; He et al., 2010). This also implies that the temperature variations and therefore the stress–strain responses of the material strongly depend on particular specimen geometry (such as the shape and size of the rod, strip or tube) and the ambient conditions (air or water of different velocities).

4. **The coupling between thermal and mechanical quantities**: This is demonstrated in Fig. 1 by comparing the heat source and the temperature profile with the stress–strain curves during the transient stage (Yin, 2013). The temperature variation (due to the release and transfer of the latent and hysteresis heat) and the stress–strain response (N.B. the area of the stress–strain hysteresis loop itself is the heat source) are strongly coupled to each other and continuously change before reaching their respective steady-state profiles.

Another important issue in the cyclic response of SMAs, one must fully recognize that the material nonlinearity (i.e. phase transition), non-equilibrium (heat transfer) and the coupling are inherent in the cyclic phase transition process and constitute the main physical origins of the observed temporal evolution of the thermal and mechanical fields. The above factors also pose challenging issues in experimental quantification of the cyclic responses of the material. Firstly, the transient temperature variations of the specimen during the cyclic phase transitions must be measured with sufficient time resolution, especially for high frequency loading. Secondly, the specific heat transfer condition of the specimen with the ambient must be well calibrated and controlled, because the tests with specimens of different size and shape in different surrounding ambient condition (defining the time scale of heat transfer to the outside) (Ortin and Delaey, 2002; He and Sun, 2010a; He et al., 2010). Thirdly, to have a good understanding of the physical processes, the measurements of the mechanical and thermal quantities need to be synchronized. Special loading and measurement facilities need to be developed to meet these challenges. Finally, in modeling and understanding this complicated process, simple method and model must be developed to get insight on the nature of the process and to identify the key governing parameters.
actual service. The second mechanism of stabilization is the thermomechanical coupling (see Fig. 1), it operates via the temperature variation of the specimen (caused by the latent heat release/absorption, hysteresis heat release and the heat transfer with the ambient) and the temperature dependence of the transition stress. For an as-received fresh specimen, this second mechanism works interactively under cyclic transformation with the first mechanism to reach the stabilized cyclic response of the material. Preliminary experiments on the trained NiTi rods where the first mechanism has reached saturation (Tobushi et al., 1998; Torra et al., 2010; He and Sun, 2010a; Soul et al., 2010; Yin, 2013) showed that the second mechanism alone can produce a distinctive frequency-dependent temperature variation, leading to the change of the stress–strain curve and the hysteresis loop area (damping capacity). Although research work on the thermomechanical coupling and cyclic behavior of NiTi have increased rapidly over the past decades (Miyazaki et al., 1986; Stachowiak and McCormick, 1988; Tanaka et al., 1992, 1993; Abeyaratne and Kim, 1997; Bo and Lagoudas, 1999; Lim and Mcdowell, 2002; Nemat-Nasser and Guo, 2007; Zaki and Moumni, 2007; Zhu and Zhang., 2007; Torra et al., 2010; Heller et al., 2009; Bernardini and Rega, 2010; He and Sun, 2010a; Morin et al., 2011a, 2011b), much work remains to be done on the role of the second mechanism in the cyclic responses of NiTi. In particular, systematic experiments and modeling which aim at revealing the roles of the heat release and the heat transfer and the associated characteristic time scales in the responses over a wide range of loading frequencies are required.

This paper reports the experiments and analysis on the time evolution of temperature and stress–strain curves of superelastic NiTi polycrystalline shape memory alloy rods during the reversible phase transitions under cyclic tensile deformation at different frequencies in stagnant air. Section 2 describes the NiTi material, the specimen and the experimental setups. Section 3 reports on the experimental results obtained from synchronized measurement of the temperature and stress–strain curves in the first 5 cycles and the steady-state (stabilized) behavior are.
discussed. In Section 4, theoretical analysis is performed to understand the phenomena and is compared with the experimental data. The conclusions are given in Section 5.

2. Material, specimen preparation and experimental setups

The material used in the experiment was a commercial superelastic polycrystalline, straight annealed, NiTi rod of 3.5 mm diameter with a dark oxide surface (Johnson Matthey Inc., USA). The overall length of the rod specimen used in the experiment was 70 mm with 15 mm clamping length at each end, so the gauge length of the specimen was 40 mm (Fig. 2). The austenite finish temperature \( A_f \) of the material is 17.1°C so the starting phase of the specimen is austenite at room temperature (25°C) and will exhibit superelastic behavior under stress. The grain size of the specimen is about 100 nm, the latent heat, isothermal hysteresis heat (i.e. the isothermal stress–strain loop area after training), heat capacity and other thermomechanical properties of the material are listed in Table 1. The coefficients \( k_f \) and \( k_r \) for the temperature dependence of the forward (A→M) and reverse (M→A) transformation plateau stresses measured by the isothermal tensile test at different constant temperatures are also listed in Table 1. All specimens used in the test were cut from the same piece of rod.

Displacement-controlled sinusoidal cyclic tensile loading and unloading was carried out on a MTS 858 universal testing machine with a hydraulic power unit. To examine the effect of frequency on the thermal and mechanical responses, 15 cyclic deformation frequencies ranging from 0.0004 Hz to 1 Hz (average strain rate \( 4.8 \times 10^{-3} / s \) to \( 1.2 \times 10^{-1} / s \)) were used in the test as shown in Table 2. The measurement of the evolutions of the stress–strain curve and the surface temperature of the specimen were synchronized in the test. Specially designed grips clamped the rod specimen to ensure that no slip occurred between the specimen and the grips during the cyclic tests (Fig. 2).

An ultra-thin naked ‘K’ type thermocouple (from Omega, USA) having a diameter of 0.025 mm and relaxation time of 0.05 s was used to capture the rapid temperature variation in the high frequency cyclic phase transition. Two very thin

<table>
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<th>Property</th>
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<th>Unit</th>
<th>Value</th>
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<td>Specific heat capacity per unit mass</td>
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<td>J/(kg K)</td>
<td>500</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>kg m(^{-3})</td>
<td>6450</td>
</tr>
<tr>
<td>Heat capacity per unit volume</td>
<td>( \lambda = \rho c_p )</td>
<td>J/m(^{3}) K</td>
<td>3.225 \times 10(^6)</td>
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<tr>
<td>Latent heat per unit volume</td>
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<td>J/m(^{3})</td>
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<tr>
<td>Isothermal hysteresis heat per unit volume</td>
<td>( D_0 )</td>
<td>J/m(^{3})</td>
<td>6.58 \times 10(^6)</td>
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<td>Temperature coefficient for isothermal forward transformation</td>
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<td>MPa/K</td>
<td>6.56</td>
</tr>
<tr>
<td>Temperature coefficient for isothermal reverse transformation</td>
<td>( k_r )</td>
<td>MPa/K</td>
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<tr>
<td>Austenite finish temperature</td>
<td>( A_f )</td>
<td>°C</td>
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</table>

Fig. 2. Schematic drawing showing the experimental set-up for the synchronized measurement of thermal and mechanical quantities.

Table 1
Thermal and mechanical properties of the polycrystalline NiTi SMA rod.
and which was measured by an independent experiment. To measure frequency on the temperature and the stress respectively. Stable temperature differences (Appendix A) and heated by electric resistance heating. The electrical current ranges from 0.2 A to 25 A to achieve different stress and a gradual increase and saturation (see Fig. 3(b), such training leads to the mechanical shake down of the stress transformation and eliminate the effect of dislocation. A preload stress of 10 MPa was applied in order to avoid unexpected connection cables of the thermocouple were electrically insulated by RTV silicon (Versachem, USA). To ensure good contact, the thermocouple was pasted on the surface of the specimen using Kapton Tape. In our experiments, it should be noticed that the localized transformation domains in NiTi lead to a non-uniform temperature field but heat conduction always tends to make the temperature uniform. This means that for most of the deformation period, the temperature field can be treated as uniform due to heat conduction (see Appendix A for more detailed analysis). To check the position-dependence in the measured temperatures, three thermocouples were located at different points along the gauge length section (Fig. 2). The results showed that the response from these three thermocouples were almost identical (Fig. 2) at the frequency of 0.0007 Hz, so the spatial inhomogeneity of the measured temperature within the gauge length can be ignored and the data from a single thermocouple can be used as the specimen’s representative temperature (i.e. volume-averaged temperature). The time evolution of the temperature was recorded by an Agilent 34970A data logger with maximum 215 readings per second. Typically more than 50 temperature data points were acquired in each cycle.

The experiments were conducted in the lab with ambient static air temperature of 25 °C. The transfer of heat between the specimen and the surroundings in the actual experiment was attributed to conduction via the grips and convection via air. The heat transfer condition of the rod specimen can be characterized by an overall lumped convective coefficient $h$ which was measured by an independent experiment. To measure $h$ under actual test conditions, the NiTi rod was inserted into two stainless steel blocks (made of the same material having the same depth of contact as the grips, see Fig. A2(b) in Appendix A) and heated by electric resistance heating. The electrical current ranges from 0.2 A to 25 A to achieve different stable temperature differences $\Delta T$ between the rod and the air ($\Delta T$ ranges from 0 °C to 127 °C). The overall lumped convective coefficient $h$ of the rod specimen consists of two terms representing the contribution from pure conduction and pure convection respectively (as shown in Eq. (A9), details discussed in Appendix A). The measured relationship between $h$ and $\Delta T$ (see Holman, 2009) is summarized in Fig. 3(a).

To remove the effect of cyclic plasticity due to dislocation (the first mechanism), the fresh as-received rod specimen was trained by cyclic deformation for 200 cycles at strain rate $1.5 \times 10^{-3}$/s with a maximum strain of 7.5% to ensure full transformation and eliminate the effect of dislocation. A preload stress of 10 MPa was applied in order to avoid unexpected compression in training. As shown in Fig. 3(b), such training leads to the mechanical shake down of the stress–strain curve with a final repeating hysteresis loop, which is evidenced by a gradual decrease and saturation ($\approx 100$ MPa) of the applied stress and a gradual increase and saturation ($\approx 1\%$) of the residual strain. With such training, the effect of the first mechanism in the subsequent cyclic tests can be ignored. Cyclic tests were then performed on the trained specimen under displacement-controlled sinusoidal deformation which gives a cyclic strain variation from 0%→6%→0% (see Fig. 2). A programmed preload of 50 MPa was set for each cyclic test to avoid possible compression in high frequency unloading in case there was an unexpected undershoot of the machine.

All the tested specimens had been trained before the cyclic tests. Moreover, Fig. 3(c) shows the stress–strain curves (at strain rate of $1.2 \times 10^{-2}$/s and with a pre-stress of 50 MPa) for a single rod specimen after each of the 9 cyclic tests. It is seen that the changes in the stress–strain curves are very small (except the one at 1 Hz), therefore the damage caused by the cyclic loading was judged as ignorable.

### 3. Experimental results

The variations of temperature and stress–strain curves due to the pure thermal effect (i.e. via the second mechanism) during cyclic loading mainly consist of two stages: transient stage and steady-state stage. The effect of the deformation frequency on the temperature and the stress–strain responses in the two stages will be reported in Sections 3.1 and 3.2 respectively.

#### Table 2
Summary of cyclic loading conditions of the 15 tests.

<table>
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<tr>
<th>Test no.</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
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<td>$1 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$4 \times 10^{-3}$</td>
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<tr>
<td>Average strain rate (s$^{-1}$)</td>
<td>$4.8 \times 10^{-5}$</td>
<td>$8.4 \times 10^{-5}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$2.4 \times 10^{-4}$</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>Test no.</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
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<tr>
<td>Frequency (Hz)</td>
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<td>$1 \times 10^{-2}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$4 \times 10^{-2}$</td>
<td>$7 \times 10^{-2}$</td>
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<tr>
<td>Average strain rate (s$^{-1}$)</td>
<td>$8.4 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$2.4 \times 10^{-3}$</td>
<td>$4.8 \times 10^{-3}$</td>
<td>$8.4 \times 10^{-3}$</td>
</tr>
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<td>Test no.</td>
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<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
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<tr>
<td>Frequency (Hz)</td>
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<td>$4 \times 10^{-1}$</td>
<td>$7 \times 10^{-1}$</td>
<td>1</td>
</tr>
<tr>
<td>Average strain rate (s$^{-1}$)</td>
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<td>$2.4 \times 10^{-2}$</td>
<td>$4.8 \times 10^{-2}$</td>
<td>$8.4 \times 10^{-2}$</td>
<td>$1.2 \times 10^{-1}$</td>
</tr>
</tbody>
</table>
3.1. Temperature and stress variations in the transient stage

Figs. 4–6 show the typical results of the measured temperature variations and non-isothermal stress–strain curves at the low, intermediate and high frequencies of 0.0007 Hz, 0.04 Hz and 1 Hz respectively. The measured temperature and stress oscillations for all 15 frequencies are summarized in Figs. 7 and 8, respectively, showing that the cyclic responses of the specimen are highly dependent on frequency. For each frequency, there exists a transient stage where the response drifts with the number of cycles until it eventually reaches the steady-state stage.
**Fig. 4.** Evolutions of (a) temperature and (b) stress–strain curve from transient to steady-state under cyclic deformation of the trained rod at the low deformation frequency of 0.0007 Hz. The synchronized stress, strain and temperature variations in a steady-state cycle are shown in (c) \(T_{\text{max}} - \text{steady-state maximum temperature, } T_{\text{min}} - \text{steady-state minimum temperature, } T_{\text{mean}} - \text{steady-state mean temperature, } \Delta T - \text{steady-state temperature oscillation amplitude.} \)

**Fig. 5.** Evolutions of (a) temperature and (b) stress–strain curve from transient to steady-state under cyclic deformation of the trained rod at the intermediate deformation frequency of 0.04 Hz. The synchronized stress, strain and temperature variations in a steady-state cycle are shown in (c) \(T_{\text{max}} - \text{steady-state maximum temperature, } T_{\text{min}} - \text{steady-state minimum temperature, } T_{\text{mean}} - \text{steady-state mean temperature, } \Delta T - \text{steady-state temperature oscillation amplitude.} \)
As shown in Fig. 4, at the very low frequency of 0.0007 Hz where the average strain rate in the cycle was only $8.4 \times 10^{-5}$/s, the transient stages of the temperature and the stress–strain curve were much less obvious. The temperature oscillated with the amplitude of $\Delta T_s = \frac{T_{\text{max}} - T_{\text{min}}}{2} = 8.0^\circ C$ in each cycle and there was no observable difference between the first cycle and the steady-state cycles. The corresponding superelastic stress–strain curves of all cycles, though non-isothermal due to the release/absorption of the latent heat and hysteresis heat, almost coincided with each other as shown in Fig. 4(b). At this low frequency, the heat effect on the cyclic thermal and mechanical responses was very limited since the heat transfer via convection and conduction was much faster than the heat generation (i.e. loading time of 714 s). The mean temperature of each cycle ($T_{\text{mean}} = 25.1^\circ C$) was only a little higher than the room temperature ($25^\circ C$). Compared with the isothermal case (see the isothermal curve with two flat plateaus in Fig. 4(b)), the stress–strain curve exhibited a little hardening and the hysteresis loop area ($D_0$) was very close to the isothermal case ($D_{\text{iso}}$) for the post-trained specimen. It is well recognized (see Muller and Villaggio, 1977; Puglisi and Truskinovsky, 2005; Petryk, 2005; Ortin, 1995; Ortin and Delaey, 2002; Planes and Manosa, 2001; Sun and He, 2008) that the isothermal hysteresis loop area (also called rate-independent hysteresis in literature) is mainly caused by microscopic level mechanical dissipative events and can be used as a characteristic property of the polycrystalline material.

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At the intermediate frequency of 0.04 Hz (see Fig. 5), the deformation period $t_p (= 1/f)$ was 25 s, close to the characteristic lumped convection time $t_c (23–60$ s, see Fig. 3(a)) of the specimen. The heat release time at this frequency was
The steady-state temperature oscillation amplitude decreased as the frequency increased. The start temperature of the second cycle was lower than that of the first cycle and the start temperature of the third cycle was lower than that of the second cycle. The start temperature continued decreasing until it reached its steady-state value. At this frequency, the start temperature of the second cycle was lower than that of the first cycle and the start temperature of the third cycle was lower than that of the second cycle. The start temperature continued decreasing until it reached its steady-state value. About 7 cycles were needed to reach the steady state. Because of the temperature dependence of the stress, the corresponding cyclic stress–strain curves became a hardening type (mainly due to self-heating caused by latent heat in loading and self-cooling in unloading). Accompanying the mean temperature decrease in the transient stage, the stress–strain curve drifted downward, eventually reaching its steady-state loop.

Fig. 7. Temperature evolutions of the trained rod specimen under cyclic deformation of 15 different frequencies (\(\Delta T^\alpha\) – steady-state temperature oscillation amplitude, \(T^\text{max}\) – steady-state maximum temperature, \(T^\text{mean}\) – steady-state mean temperature).
At the frequency of 1 Hz (see Fig. 6), the forward phase transition time \( t_p/2 \) was only about 0.5 s which was much shorter than the characteristic lumped convection time \( t_L^c \) of the specimen (23–60 s as shown in Fig. 3(a)). The thermal effect on the cyclic responses became even more significant (Fig. 6). On the one hand, the fast release/absorption of the...
latent heat caused a very rapid temperature increase/decrease of the specimen and made the temperature oscillation amplitude to reach the saturation value of 23 °C. On the other hand, the released heat from the hysteresis loop (mechanical dissipative heat) in each loading–unloading cycle had insufficient time to transfer out (because $t_p \ll T_{\text{transient}}$) and rapidly accumulated in the specimen. This hysteresis heat accumulation caused a large increase in the mean temperature ($T_{\text{mean}}$) of the specimen and a significant shift up of the stress–strain loops. As shown in Fig. 6(a), $T_{\text{mean}}$ increased to a saturation plateau of 60.9 °C. The saturation was due to a balance between the hysteresis heat release and the heat transfer to environment. During the transient stage, the increase of $T_{\text{mean}}$ also increased the amount of heat transferred out (because heat convection increases with $T_{\text{mean}} - T_0$). In the steady-state stage, the heat transferred out equaled to the hysteresis heat released so there was no further heat accumulation inside the specimen and $T_{\text{mean}}$ did not increase further (i.e. saturation).

The amplitude of the temperature oscillation in the transient stage was almost constant for all cycles. The little change in the temperature oscillation amplitude shown in Fig. 6(d) ($\Delta T = 20.6$ °C for the 1st cycle while $\Delta T = 23.0$ °C for the steady-state cycle) was due to the undershoot of the machine (in the starting cycles the maximum strain did not reach the prescribed value (6%)). The peak value of the temperature ($T_{\text{max}}$) increased from 46.5 °C in the first cycle to 72.4 °C in the steady-state cycles. It took about 110 cycles to reach the steady-state. The corresponding stress–strain curves that exhibited strong hardening due to rapid temperature increase/decrease in loading/unloading shifted upward significantly and eventually reached a stable loop because of the repeated temperature variation in the steady-state and the temperature dependence of the transformation stress. It is noticed that the steady-state non-isothermal hysteresis loop area at 1 Hz is much smaller than the isothermal case and no longer represents a characteristic property of the material (see He et al., 2010; He and Sun, 2011 on the rate dependent hysteresis loop area).

Figs. 7 and 8 summarize the variations of temperature and stress with time for all the cyclic tests at 15 different frequencies. For all the cases, the temperature variation with time at each frequency can be treated as the superposition of an oscillation part over a mean temperature part. The amplitude of the temperature oscillation increased monotonically with the frequency and finally reached a saturated value (see the cases of 0.4 Hz, 0.7 Hz and 1 Hz in Fig. 7). The mean temperature in a cycle ($T_{\text{mean}}$) was always higher than the room temperature $T_0$ (= 25 °C). It drifted down with the number of cycles for the intermediate frequency deformation (see the cases from 0.004 Hz to 0.1 Hz in Fig. 7) and drifted up for high frequency deformation (see the cases from 0.2 Hz to 1 Hz in Fig. 7) until the steady-state ($T_{\text{mean}}$) was reached. The number of cycles needed to reach the steady-state for each frequency of loading varied with the loading frequency. As will be discussed in Section 4, the time scale of the transient stage is related to the characteristic heat transfer time of the system (i.e. the specimen and the ambient). The difference of the maximum temperatures between the first and steady-state cycles ($\Delta T_{\text{max}} = T_{\text{max}} - T_{\text{1st}}$) varies non-monotonically with the frequency as shown in Fig. 9. We can see the same trend in the difference of the mean-temperature between the first and the steady-state cycles ($\Delta T_{\text{mean}} = T_{\text{mean}} - T_{\text{1st}}$) as plotted in Fig. 9 for comparison (N.B. in the experiment, we used the mean temperature over half cycle to have more data points and to better describe the transient stage temperature change).

For each frequency, the measured stress–strain curves changed with the number of cycles in the transient stage in a similar way to that of the temperature. Compared with the first cycle, the stress–strain loops changed little in the low frequency range (see the typical curves at 0.0007 Hz in Fig. 4), drifted down in the intermediate frequency range (see the typical curves at 0.04 Hz in Fig. 5) and drifted up significantly in the high frequency range (see the curves at 1 Hz in Fig. 6). The difference in the maximum stresses between the first cycle and the steady-state cycle $\Delta \sigma_{\text{max}} = \sigma_{\text{max}} - \sigma_{\text{1st}}$ for all frequencies are marked in Fig. 8 and summarized in Fig. 9 for comparison with the temperature change. It can be seen that, with the increase in frequency, the values of $\Delta \sigma_{\text{max}}$, $\Delta \sigma_{\text{mean}}$ and $\Delta \tau_{\text{mean}}$ changed non-monotonically from the negatives in the low and intermediate frequency range to the positives in the high frequency range. In addition to the extremely low frequency where it is isothermal (a trivial case here), there exists a critical frequency of about 0.2 Hz (with the deformation period $t_p = 5$ s) at which $\Delta \sigma_{\text{max}}$, $\Delta \sigma_{\text{mean}}$ and $\Delta \tau_{\text{mean}}$ are zero and for which there is no transient stage in the temperature and stress–strain profiles. The roles of the two time scales (loading time and heat transfer time) in the frequency-dependent temperature and stress oscillations will be discussed in Section 4.

### 3.2. Temperature and stress variations in the steady-state stage

In the steady-state stage, neither the temperature nor the stress–strain variations changed with the number of cycles during the cyclic phase transition. The temperature simply oscillated with a constant amplitude $\Delta T = T_{\text{max}} - T_{\text{min}}$ around a mean temperature plateau ($T_{\text{mean}}$), making it easily quantified by the superposition of these two parts. The stress oscillation can be characterized by the maximum stress $\sigma_{\text{max}}$ since the minimum strain was selected to make the minimum stress approximately zero. The existence of the steady-state is indicative of the fact that the total heat generated in a cycle inside the specimen is equal to the heat transferred to the ambient (Section 4 provides a detailed analysis).

The variation of the steady-state mean temperature $T_{\text{mean}}$ (i.e. the value of the plateau) with the frequency is shown in Fig. 10(a), where $T_{\text{mean}}$ increases monotonically with the frequency. In particular, $T_{\text{mean}}$ increases more rapidly in the high frequency range because of the high rate of hysteresis heat accumulation at those frequencies. The amplitude of temperature oscillation in the steady-state also increases monotonically with the frequency as shown in Fig. 10(b), but it became saturated (around 23 °C) in the high frequency range. The oscillation comes from the release and absorption of the same amount of latent heat ($l_0 = 7.74 \times 10^7$ J/m$^3$) so the amplitude of the oscillation in the high frequency (near adiabatic
condition) became constant. The maximum and the minimum temperatures in the steady-state cycle ($T_{\text{max}}^{\text{st}}$ and $T_{\text{min}}^{\text{st}}$) are plotted in Fig. 10(c). It is seen that $T_{\text{max}}^{\text{st}}$ (due to the forward transformation) increases monotonically with the frequency while $T_{\text{min}}^{\text{st}}$ (due to the reverse transformation) varies non-monotonically with the frequency. This type of non-monotonicity
Fig. 10. Experimental results of the frequency dependence of temperature variation in the steady-state cycles: (a) the mean temperature $T_{\text{mean}}$, (b) the temperature oscillation amplitude $\Delta T_s$ and (c) the maximum temperature $T_{\text{max}}$ and the minimum temperature $T_{\text{min}}$. The modeling results from Section 4 are also plotted for comparison.
in $T_{\text{min}}$ is similar to that found in a single loading–unloading cycle under different constant strain rates and can be explained and captured by the model prediction in the work of He and Sun (2010a, 2011).

Due to the monotonic increase of $T_{\text{max}}$ with the frequency, the maximum stress $\sigma_{\text{max}}$ in the steady-state stress–strain curve also increased monotonically with the frequency as shown in Fig. 11(a). The correlation between $\sigma_{\text{max}}$ and $T_{\text{max}}$, shown in Fig. 11(b), indicates a linear relationship with a proportional coefficient of 4.5 MPa/oC which is lower than the measured temperature coefficient $k_f \approx 6.56$ MPa/oC of the isothermal forward and reverse transformation plateau stresses shown in Fig. 11(c). We have to emphasize that, since the real phase transition in loading is a rather complicated process involving strong non-isothermal and non-homogeneous temperature distribution, we can only roughly estimate the stress oscillation from the temperature oscillation
data based on such correlation. In the present experiment, a cyclic transformation at the frequency of 1 Hz in room temperature (\(=25\ ^\circ\text{C}\)) stagnant air can raise the specimen’s temperature to 72.5 \(^\circ\text{C}\) and the stress up to 611 MPa (see Fig. 6). Although large increase in the stress due to the latent heat at relatively adiabatic loading rates was reported by others, no systematic measurement and analysis of this kind of large increase in the stress due to the transformation hysteresis heat and the latent heat has been performed before.

Fig. 12 shows the non-monotonic frequency dependence of the measured hysteresis loop areas of the steady-state cycle (\(D_s\)). It is seen that \(D_s\) first increased from the isothermal value of \(6.61 \times 10^6 \text{J/m}^3\) to the peak value \(7.99 \times 10^6 \text{J/m}^3\) (at 0.002 Hz) then decreased to \(3.61 \times 10^6 \text{J/m}^3\) (at 1 Hz). \(D_s\) represents the mechanical energy dissipation per unit volume per steady-state cycle and it is an important parameter for SMA damping applications, e.g., seismic dampers. It is also an important factor governing the mean temperature of the steady-state cycle (\(T_{\text{mean}}\)). Such non-monotonic variation of \(D_s\) with the frequency will be analyzed and discussed in Section 4.

Finally, it must be emphasized that after each cyclic experiment of the trained specimen, the residual strain was checked and found to be ignorable. We reloaded the specimen at a low strain rate for one cycle after each test and found the specimen’s stress–strain curves to be almost the same (Fig. 3(c)), which means that the microstructure change can be ignored after these cyclic tests. This confirmed the pre-assumption that the thermo-mechanical response observed in the above cyclic deformation is dominated by the second mechanism (thermal effect) and we can basically exclude the effect of plasticity/dislocation and residual martensite on the observed phenomena in this section. (Remark: In general there should be no exclusive operation between the first and the second mechanisms in the cyclic phase transition. They both can produce the transient and steady-state responses during the cyclic deformation. Here, in order to clearly differentiate the contributions from the two different mechanisms, we purposely trained the specimen to remove the major effects of dislocation/plasticity and residual martensite so as to clearly illustrate the pure thermal effect from the second mechanism).

To end this section, we summarize the key observations on the thermomechanical responses of the NiTi rods under cyclic phase transition as follows:

1. Under cyclic deformation both temperature and stress vary in the form of periodic oscillations which experience transient stages before reaching their steady-states (thermal shake down). The time \(t_{\text{transient}} (=N_{\text{transient}}/f)\) required to reach the steady-state cycle appears proportional to the lumped convection time \(t_h\) by a factor of 3 (see Fig. 13). The trend and the magnitude of the temperature variation in the transient stage strongly depends on the frequency (Fig. 7): both the mean temperature and the maximum temperature of a cycle decrease at relatively low frequency and increase at high frequency with the cycle number in the transient stage. The mean temperature variation in the transient stage vanishes at the critical frequency of about 0.2 Hz (see Figs. 7, 9 and 14).

2. Following the transient stage is the stage of steady-state oscillation where the temperature oscillates around a mean temperature plateau for each given frequency (Fig. 7). The oscillation is mainly attributed to the release/absorption of the latent heat and the amplitude of the oscillation increases with the loading frequency and eventually becomes saturated at the high frequency range (near the adiabatic condition, see Fig. 10(b)). The value of the steady-state mean temperature increases monotonically with the frequency (Fig. 10(a)). Such increase is attributed to the accumulation of the hysteresis heat (steady-state stress–strain loop area). It is observed that the higher the frequency, the higher the mean temperature plateau. The steady-state mean temperature plateau at a given frequency reflects the balance between the heat supply rate and the heat transfer rate, which is determined by the loading frequency (setting the time scale of heat release), the characteristic hysteresis heat, heat capacity and the convective heat transfer property of the ambient.
Under cyclic deformation, the stress–strain curve varied at the same pace as the temperature because of the intrinsic temperature dependence of the stress (Claperyon–Clausius relation). The stress–strain response also experienced transient stage where the stress–strain curve gradually drifts downward or upward with the number of cycles before it reaches the steady-state hysteresis loop (Figs. 8 and 9). As summarized in Fig. 9, the frequency dependence of stress follows the same trend as that of the temperature. Also, the maximum stress and the maximum temperature in the steady-state correlated linearly. There is significant increase in the stress due to the transformation heat (hysteresis heat plus latent heat). It is believed that such heat effect, which is ignored in most of the fatigue and cyclic phase transition of NiTi, will be an important factor in determining the cyclic response of the material and will lead to the acceleration of degradation and fatigue failure of the material under high frequency loading conditions.

4. Analysis and discussion

To have a deep understanding of the above observed phenomena, especially to quantify the roles of the time scale of heat release and the time scale of the heat transfer in the observed material behavior, we need to analyze both transient and steady-state responses to identify the governing parameters. It is important to note that, as shown in Fig. 1, in the transient stage, the stress–strain curve (N.B. its area represents the mechanically dissipated heat (heat source $D$)) and the temperature variation in a cycle are coupled to each other and keep changing until the steady-state is reached. The lumped heat transfer equation (Eq. (A10) in Appendix A) for the temperature history will be a nonlinear ordinary differential equation since both the lumped convection coefficient $\bar{h}$ and the hysteresis heat $D$ are not constants and depend on the frequency and number of cycles. Instead of solving this complex nonlinear equation, we adopt simplified assumptions for $\bar{h}$ and $D$ in the following

![Fig. 13. Comparison among the frequency dependent lumped relaxation time $\bar{\tau}$, the time needed to reach steady-state ($t_{\text{transient}}$), and $3.2\bar{\tau}$.

![Fig. 14. Comparison between the experimental results and model predictions (Eq.(6)) of the mean temperature change $\Delta T_{\text{transient}}^{\text{mean}}$ in the transient stage. The measured critical frequency ($f_{\text{critical}}^{\text{exp}}$) and the predicted critical frequency ($f_{\text{critical}}^{\text{model}}$) are also shown for comparison.](image-url)
(also see Appendix A) and aim to capture the key quantitative features of the transient and the steady-state temperature variations under different frequencies in the ambient static air.

Consider a NiTi SMA rod specimen of radius \( R \) and gauge length \( L_0 \) under displacement-controlled sinusoidal cyclic deformation of frequency \( f \) (the period \( t_p = 1/f \)). The specimen’s end displacement \( u(t) \), nominal strain \( \epsilon(t) \) and strain rate \( \dot{\epsilon}(t) \) for \( t \in [0, \infty) \) are respectively:

\[
\begin{align*}
\dot{u}(t) &= \frac{u_0}{2}(1 - \cos \omega t) \\
\epsilon(t) &= \frac{u_0}{2L_0}(1 - \cos \omega t) \\
\dot{\epsilon}(t) &= \frac{u_0 \omega}{2L_0} \sin(\omega t)
\end{align*}
\]

where \( u_0 \) is the maximum end-displacement and \( \omega \) is the angular frequency (\( \omega = 2\pi f \)). By using the lumped-analysis method (Cotta and Mikhailov, 1997) and certain simplifying assumptions (see Appendix A), we derived the following heat transfer governing equation for the rod specimen’s volume-averaged temperature \( T(t) \) as

\[
\lambda \frac{dT(t)}{dt} = -\frac{2\bar{h}}{R}(T(t) - T_0) + \frac{l_0 \omega}{2 \pi} \sin(\omega t) + \frac{D_0 \omega}{\pi} \sin^2(\omega t)
\]

where \( \lambda \) is the heat capacity per unit volume, \( \bar{h} \) is the lumped convective coefficient of the rod (including convection and conduction). The first term of the right-hand side of Eq. (4) represents the lumped heat convection while the second and third terms represent the rate of the latent heat release and the hysteresis heat release, respectively. The rates of the two heat sources are approximated in different sinusoidal forms (see Appendix A for details). For each given loading frequency, both \( \bar{h} \) and \( D \) depend on the specimen’s temperature history and therefore are not constants in the transient stage. For the purpose of simplicity and as a first approximation, \( \bar{h} \) and \( D \) can take their respective steady-state values (i.e. let \( \bar{h} = \bar{h}_s \) and \( D = D_s \) at a given loading frequency \( f \)) in Eq. (4). Then Eq. (4) can be easily solved as a linear ODE with the initial condition \( T(t)|_{t=0} = T_0 \) to give the expression of \( T(t) \) as

\[
T(t) = T_0 + A + B e^{-\frac{t}{\tau_c}} + C \sin(\omega t - \alpha_1) - E \sin(2\omega t + \alpha_2)
\]

where \( A, B, C, \alpha_1, \) and \( \alpha_2 \), as shown below, are combinations of the internal and external governing parameters which characterize the material properties, the ambient and the loading:

\[
\begin{align*}
A &= \frac{D_0 \tau_c}{\pi} = \frac{D_0 \tau_c}{\pi} = \frac{D_0 \tau_c}{\pi} \\
B &= \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} \\
C &= \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} \\
E &= \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} = \frac{l_0 \omega}{\pi} \\
\alpha_1 &= \cos^{-1}\left(\frac{1}{\sqrt{1 + l_0 \omega^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1 + l_0 \omega^2}}\right) \\
\alpha_2 &= \sin^{-1}\left(\frac{1}{\sqrt{1 + l_0 \omega^2}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{1 + l_0 \omega^2}}\right)
\end{align*}
\]

In the above, the loading period \( t_p \) has been normalized by the characteristic lumped convection time \( \tau_c \) (i.e. \( \tau_c = \frac{t_p}{\bar{h}_s} \)). Since \( l_0 > D \) for most NiTi polycrystals (He and Sun, 2010a), we have \( C > E \) and the last term of Eq. (5b) can be ignored. So the temperature evolution \( T(t) \) (or the temperature difference \( \Delta T(t) = T(t) - T_0 \)) is mainly governed by three terms:

\[
T(t) = T_0 + A + B e^{-\frac{t}{\tau_c}} + C \sin(\omega t - \alpha_1)
\]
Eq. (5c) can be considered as the superposition of the mean temperature evolution \( T_{\text{mean}}(t) \) and the temperature oscillation \( T_{\text{oscillation}}(t) \):

\[
T(t) = T_0 + A + Be^{-t/T_{\text{mean}}} + C \sin(\omega t - \alpha) = T_{\text{mean}}(t) + T_{\text{oscillation}}(t),
\]

where

\[
\begin{aligned}
T_{\text{mean}}(t) &= T_0 + A + Be^{-t/T_{\text{mean}}} \\
T_{\text{oscillation}}(t) &= C \sin(\omega t - \alpha)
\end{aligned}
\]  

(5d)

It is noted that the term \( B e^{-t/T_{\text{mean}}} \) characterizes the mean temperature change in the transient stage (Fig. 14), \( A + T_0 \) is the steady-state mean temperature and \( C \) is the amplitude of oscillation (Fig. 10). In the following, we discuss in detail the solutions for the transient and steady-state stages and compare the solutions with the experimental data.

4.1. Predictions of temperature variation in the transient stage

To understand the measured temperature change \( \Delta T_{\text{mean}}^{\text{transient}} \) in the transient stage as shown in Fig. 9, we examine the following theoretical predictions of Eq. (5d):

\[
\Delta T_{\text{mean}}^{\text{transient}} = (T_{\text{mean}})_t = \infty - (T_{\text{mean}})_t = 0 = B \left[ e^{-t/T_{\text{mean}}} - e^{-t/T_{\text{mean}}} \right] = -B
\]

\[
= - \left[ \frac{l_0}{2 \lambda (1 + (T_p^2/4\pi^2))} - \frac{4D_s}{\lambda \cdot T_p^2 \cdot ((2\pi^2/4\pi^2) + 4)} \right] = \left[ \frac{l_0}{2 \lambda} \right] \frac{4\pi^2 f^2 \cdot \frac{T_{\text{oscillation}}^2}{h}}{(4\pi^2 f^2 \cdot \frac{T_{\text{oscillation}}^2}{h} + 1)} \left[ \frac{8D_s}{l_0} \frac{1}{(16\pi^2 f^2 \cdot \frac{T_{\text{oscillation}}^2}{h} + 1)} - 1 \right]
\]  

(6)

It shows that \( \Delta T_{\text{mean}}^{\text{transient}} \) originates from the latent heat \( l_0 \) and the hysteresis heat \( D_s \). Two types of sources have opposite effects on \( \Delta T_{\text{mean}}^{\text{transient}} \) and the sign of \( \Delta T_{\text{mean}}^{\text{transient}} \) is frequency dependent through the term \( \frac{1}{2\lambda} \). The opposite effects of \( l_0 \) and \( D_s \) on \( \Delta T_{\text{mean}}^{\text{transient}} \) can be explained as follow: when considering only the effect of the hysteresis heat \( D_s \) (assuming \( l_0 = 0 \) in Eq. (5a)), we see that the temperature at the end of the 1st loading cycle will be higher than the start temperature of the cycle (i.e. \( T_{\text{1st End}}^{\text{Start}} = T_{\text{1st Start}}^{\text{Start}} \)), except the isothermal case where \( T_p = \infty \) (i.e. \( T_{\text{mean}}^0 \)). The opposite effects of \( l_0 \) and \( D_s \) on \( \Delta T_{\text{mean}}^{\text{transient}} \) can be explained as follow: when considering only the effect of the hysteresis heat \( D_s \) (assuming \( l_0 = 0 \) in Eq. (5a)), we see that the temperature at the end of the 1st loading cycle will be higher than the start temperature of the cycle (i.e. \( T_{\text{1st End}}^{\text{Start}} = T_{\text{1st Start}}^{\text{Start}} \)), except the isothermal case where \( T_p = \infty \) (i.e. \( T_{\text{mean}}^0 \)).

Using typical values of the tested NiTi material (\( l_0 \), \( \lambda \) in Table 1, \( T_p \) in Fig. 3(a) and the hysteresis heat \( D_s \) in Fig. 12), the predicted frequency dependence of \( \Delta T_{\text{mean}}^{\text{transient}} \) is shown in Fig. 14. It is seen that, depending on the frequency, the predicted \( \Delta T_{\text{mean}}^{\text{transient}} \) can be positive and negative and varies non-monotonically with the frequency. The predictions agree well with the experimental data. Theoretically, there are two frequencies for which \( \Delta T_{\text{mean}}^{\text{transient}} = 0 \) (i.e. \( B = 0 \)). The first is a trivial case where \( f = 0 \) Hz (i.e. \( T_p = \infty \)) since the effects of the heat sources on the temperature vanish. The second case of \( \Delta T_{\text{mean}}^{\text{transient}} = 0 \) at \( f = 0.23 \) Hz is of particular interest where the effects of latent heat and hysteresis heat cancel each other (see Eq. (6)), which was experimentally observed at a frequency around \( f = 0.2 \) Hz. For these two cases (with \( \Delta T_{\text{mean}}^{\text{transient}} = 0 \), there is no transient stage, i.e., the temperature variation and the stress–strain loops of all cycles coincide (see the experimental data of \( f = 0.1 \) Hz and \( f = 0.23 \) Hz in Figs. 7 and 8). For all other frequencies (\( f \neq 0 \) Hz and \( f \neq 0.23 \) Hz), there always exists transient stages before the steady-states are reached. For each of these frequencies, the time \( t_{\text{transient}} \) required to reach the steady-state can be roughly estimated by the
following truncation:

$$\frac{(T_{\text{mean}})_t = t_{\text{transient}} - (T_{\text{mean}})_0 - (T_{\text{mean}})_{t \to \infty}}{\frac{e^{-t_{\text{transient}}/t_p}}{e^{-t_0/t_p}}} = 4\% \Rightarrow t_{\text{transient}} = \frac{t_p}{4\%} \ln \left( \frac{1}{4\%} \right) = 3.2\frac{t_p}{h}$$

(7)

That means, approximately, it takes about 3 times of the characteristic heat convection time $t_p$ to reach the steady-state. The experimentally observed $t_{\text{transient}}$ versus the frequency is shown in Fig. 13 where the measured $t_p$ versus the frequency is also plotted for comparison. It is seen that, for high frequencies ($f > 0.1$ Hz), $t_p$ is of the order of $\approx 30$ s and $t_{\text{transient}}$ is of the order of $\approx 100$ s which is indeed around 3 times of $t_p$. The number of cycles to reach the steady-state can be approximated as a linear function of frequency:

$$N_{\text{transient}} = \frac{t_{\text{transient}}}{t_p} \approx 3.2 \cdot \frac{t_p}{t_p} = 3.2 \cdot \frac{t_p}{f}$$

(8)

In other words, the higher the frequency, the more cycles ($N_{\text{transient}}$) are needed to reach the steady state. This is also supported by the experimental data.

4.2. Predictions of temperature variation in steady-state cycles

When $t \to \infty$, $Re^{-t/t_p} \to 0$, Eq. (5c) gives the temperature variation in the steady-state as

$$T(t) = T_0 + A + C \sin (\omega t - \alpha)$$

(9)

With the above steady-state solution, the steady-state mean temperature $T_{\text{mean}}^\circ$, temperature oscillation amplitude $\Delta T^\circ$, maximum temperature $T_{\text{max}}^\circ$ and minimum temperature $T_{\text{min}}^\circ$ can be predicted as

$$T_{\text{mean}}^\circ = T_0 + A = T_0 + \frac{D_s}{\lambda \cdot t_p}$$

(10a)

$$\Delta T^\circ = 2C = \frac{l_0}{\lambda \sqrt{1 + \left( \frac{t_p}{2\pi} \right)^2}}$$

(10b)

$$T_{\text{max}}^\circ = T_{\text{mean}}^\circ + \frac{\Delta T^\circ}{2} = T_0 + A + C = T_0 + \frac{D_s}{\lambda \cdot t_p} + \frac{l_0}{2 \lambda \sqrt{1 + \left( \frac{t_p}{2\pi} \right)^2}}$$

(10c)

$$T_{\text{min}}^\circ = T_{\text{mean}}^\circ - \frac{\Delta T^\circ}{2} = T_0 + A - C = T_0 + \frac{D_s}{\lambda \cdot t_p} - \frac{l_0}{2 \lambda \sqrt{1 + \left( \frac{t_p}{2\pi} \right)^2}}$$

(10d)

These theoretical predictions clearly show that the steady-state mean temperature $T_{\text{mean}}^\circ$ (i.e. the temperature plateaus shown in Fig. 7) is governed by the hysteresis heat $D_s$ and $t_p$ (see Eq. (10a)), while the temperature oscillation amplitude $\Delta T^\circ$ is governed by the latent heat $l_0$ and $t_p$ (Eq. (10b)). As shown in Fig. 10, these theoretical predictions agree well with the experimental data without any fitting parameters. It is seen that $T_{\text{mean}}^\circ$, $\Delta T^\circ$ and $T_{\text{max}}^\circ$ all increase monotonically with the loading frequency $f$ (decreasing $t_p$). The main difference is that, at high frequency ($t_p \to 0$), $T_{\text{mean}}^\circ$ and $T_{\text{max}}^\circ$ increase significantly while $\Delta T^\circ$ approaches a saturated value:

$$\lim_{t_p \to 0} (\Delta T^\circ) = \frac{l_0}{\lambda}$$

(11)

It is also noted that $T_{\text{min}}^\circ$ has non-monotonic frequency dependence. This is caused by the opposite effects of the latent heat and hysteresis heat on $T_{\text{min}}^\circ$: during unloading (reverse transformation) latent heat leads to cooling while hysteresis heat leads to heating (see Eq. (10d)). A similar prediction on $T_{\text{min}}^\circ$ was also obtained by He and Sun (2010a, 2011) using a five-stage piece-wise linear stress–strain curve of the material.

The reason why the experimental results of steady-state temperature oscillation are less than the model prediction in the intermediate frequency range (0.007–0.07 Hz in Fig. 10(b)) is because at that frequency range, the transformed martensite cannot be totally transformed back to austenite after unloading due to cooling. As shown in Fig. 15, in the intermediate frequency range, the specimen’s average temperature during steady-state cyclic loading can be as low as 16.8 °C which is less than the austenite finish temperature ($A_f = 17.1$ °C). So, part of the martensite cannot be transformed back to Austenite, which is evidenced by the stress–strain responses in Fig. 15 where when the nominal strain goes back to 0, the nominal stress is less than the preload (or when the nominal stress is reduced to the preload, the strain is still not zero), indicating that residual martensite exists in the specimen. Since the amount of phase transformation becomes less, the released and
absorbed latent heat becomes less accordingly, the temperature oscillation becomes less than the model prediction (which assumes the latent heat release is constant).

The above discussion about the steady-state cycles implies that a superelastic SMA component under cyclic loading usually works at temperatures which are quite different from the ambient temperature. Particularly at high frequency, the working temperature of the SMA material can be very high due to the rapid accumulation of hysteresis heat (e.g. $T_{\text{max}} = 72.4{ }^\circ C$ for 1 Hz for the current tests), which will lead to a very high working stress of the material under displacement-controlled cyclic deformation. Finally, comparisons between the model prediction and the experiments of the whole temperature history for the three typical frequencies are shown in Fig. 16. It is seen that the predictions agree well with the experiments without any fitting parameters.

4.3. Frequency dependence of hysteresis loop area and stress responses

Using the solution of the temperature evolution (Eq. 5a) and the Clausius–Claperyon relationship, we can approximate the stress variation with time as

$$\sigma(t) = \sigma_0(T_0) + \Delta \sigma \Delta T(t)$$

Then the hysteresis loop area $D_n$ (i.e. the mechanical energy dissipation per unit volume of the $n$th cycle) can be obtained by integrating the stress in Eq. (12) over the strain for the $n$th cycle:

$$D_n = \int \sigma(t) \Delta T(t) \, dt = \int \sigma(T_0) \Delta T(t) \, dt + \int \Delta \sigma \Delta T(t) \, dt$$

$$= D_0 + \int \sigma_0(T_0) \Delta T(t) \, dt + \int \Delta \sigma \Delta T(t) \, dt$$

$$= D_0 + \int \sigma_0(T_0) \Delta T(t) \, dt + \int \Delta \sigma \Delta T(t) \, dt$$

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$$= D_0 + \int \sigma_0(T_0) \Delta T(t) \, dt + \int \Delta \sigma \Delta T(t) \, dt$$

The hysteresis loop area $D_n$ in the steady-state cycle can be easily obtained using the steady-state temperature, then Eq. (13) gives

$$D_n = D_0 + \left( k_f - k_r \right) \frac{\varepsilon_{\text{max}} D_h \omega}{2\pi} \left( k_f + k_r \right) \frac{\varepsilon_{\text{max}} \Delta T_{\text{os}}}{8 \lambda (1 + \bar{v}^2 \omega^2)}$$

where $k_f$ and $k_r$ are the temperature coefficients for forward and reverse transformation plateau stresses respectively.

Eq. (14a) shows that $D_n$ consists of three terms. The first term is the isothermal contribution $D_0$ which is a constant. The second term represents the contribution by the mean temperature change due to the hysteresis heat $D_n$. Since $k_f < k_r$, this term monotonically decreases with the frequency. The third term represents the non-isothermal contribution which originates mainly from the stress variation due to the temperature oscillation controlled by $D_h$. The predicted values of $D_n$ for cyclic loading at different frequencies are shown in Fig. 12 for comparison with the experimental data. It is seen that the measured $D_n$ first increased from $6.61 \times 10^6$ J/m$^3$ (at near isothermal case) up to the peak value $7.99 \times 10^6$ J/m$^3$ at 0.002 Hz, then decreases to $3.61 \times 10^6$ J/m$^3$ at 1 Hz. Although the prediction (predicted peak value is $9.48 \times 10^6$ J/m$^3$) is about 15% higher than the experimental data, the qualitative feature of the non-monotonic variation with frequency is well captured by the prediction (Eq. (14b)). There are two primary sources which cause the 15%
discrepancy: The first one is the approximation of the heat sources in evaluating the temperature variation. In Eq. (A7) of the Appendix A, we assume that the rate of latent heat release is proportional to the strain rate, while the hysteresis heat release rate is proportional to the square of the strain rate. These two assumptions are only correct in the lumped sense, i.e. the total

![Graphs showing temperature variations](image_url)

Fig. 16. Comparison between the experimental results and the model predictions of the temperature variations for cyclic deformation at (a) 0.004 Hz, (b) 0.04 Hz and (c) 0.4 Hz.
heat release/absorption are correct (see the discussion in the paragraph above the Eq. (A7)). Nevertheless, it is obvious that during the elastic deformation of austenite and martensite phases, there is no release of latent heat and hysteresis heat. This assumption causes error in the prediction of the temperature. The second source of error in the theoretical model is the application of the Clausius–Clapeyron relation (i.e. the temperature dependence of the transformation stresses) to the whole strain range of the loading–unloading cycle to estimate the hysteresis loop $D$ (see Eq. (13)). In fact, the cycle includes elastic loading/unloading of Austenite, elastic loading/unloading of Martensite, $A \rightarrow M$ forward transformation region and $M \rightarrow A$ reverse transformation region. The Clausius–Clapeyron relation describing the temperature dependence of phase-transformation stresses is only valid for the $A \rightarrow M$ and $M \rightarrow A$ regions where martensite and austenite phases co-exist, it is not valid for the two regions of the pure elastic deformation of austenite and martensite. That means, in these two elastic regions, even there are temperature variations, the stress variation does not follow the Clausius–Clapeyron relation and has little contribution to the hysteresis loop. Therefore, by using the Clausius–Clapeyron relation in the whole strain range of the loading–unloading cycle (i.e. phase transition regions plus elastic regions), we overestimated the temperature effect on the hysteresis loop area and therefore causing the discrepancy between prediction and measurement.

Remark: $k_f < k_c$ is generally true for most of superelastic NiTi SMAs tested without plastic deformation (see data in Fig. 11 (c) and Shaw and Kyriakides, 1995, 1997). Moreover, Liu et al. (2008) explained this relation by thermodynamic analysis with the fact that the transformation strain of the reverse transformation is smaller than that of the forward transformation (because $\frac{d \sigma}{dT}_{\text{forward-transition}} = \left(\rho \frac{\Delta S}{\Delta T_r} \right)_{\text{forward-transition}}$, $\frac{d \sigma}{dT}_{\text{reverse-transition}} = \left(\rho \frac{\Delta S}{\Delta T_r} \right)_{\text{reverse-transition}}$). Therefore, we adopt this relation $k_f < k_c$ in discussing Eq. (14a) which captures the experimental observation that $D_o$ decreases with increasing frequency in the high frequency range (Fig. 12). If we use the relationship of $k_f > k_c$ which so far has neither been observed experimentally nor proved theoretically, the results would be qualitatively different: $D_o$ would increase with increasing frequency in the high frequency range (see Eq. (14a) where the third terms approach a constant), which is not consistent with the experimental observation.

The significant frequency dependence of the cyclic stress response can be clearly seen from the $\sigma$–$e$ curves in Figs. 4–6. For 1 Hz loading, the maximum stress $\sigma_{\text{max}}^\epsilon$ reached 611.22 MPa (see Fig. 8) and maximum temperature reached $T_{\text{max}}=72.4$ °C (see Fig. 7). We can imagine that both $\sigma_{\text{max}}^\epsilon$ and $T_{\text{max}}$ would further increase if the loading frequency goes beyond 1 Hz. The significant increase in $\sigma_{\text{max}}^\epsilon$ is due to the significant increase in the mean temperature plateau $T_{\text{mean}}$ as shown in Fig. 6. So far, the maximum stress in our cyclic tests has been kept below the material’s plastic yielding stress of about 1000 MPa (see Qian et al., 2004, 2006). Here we mean two kinds of “plasticity” under external loading. One is the material’s macroscopic overall plastic flow stress which, for the NiTi used in the paper, is typically about 900–1000 MPa as measured in uniaxial tensile tests (Qian et al., 2006). The other is the microscale, discrete and local plasticity such as dislocation accumulation at the grain boundary. Such micro-plasticity events take place usually at applied stress level ($\sim$400 MPa in this manuscript during training) much lower than the macroscopic plastic yielding stress of the material, and is responsible for the observed cyclic training effect (or called micro-shakedown phenomena) in NiTi SMA. During the training process, the cyclic forward and reverse motion of the $A$–$M$ interfaces inside a grain creates dislocations that accumulate at the grain boundaries, leading to the macroscopic irreversible residual strain of the order of less than 1%. It is expected that at a higher frequency ($>1$ Hz) the first mechanism (microstructure degradation) would participate and interact with the second mechanism (thermal effect) in the cyclic deformation, which is very important but out of the scope of this paper.

5. Summary and conclusions

The thermomechanical responses of a superelastic NiTi SMA rod under displacement-controlled cyclic phase transition are rather complicated due to the microstructure degradation, temperature variation, heat transfer and the thermomechanical coupling. The cyclic stress–strain curves are non-isothermal and strongly influenced by the combination and interaction of internal and external governing parameters (internal parameters such as heat capacity, latent heat and isothermal hysteresis heat; and external parameters such as the training process, the imposed strain amplitude, deformation frequency, specimen geometry and the heat transfer condition of the ambient). The stress–strain relationship and the damping capacity are no longer pure intrinsic material properties. In this paper, to investigate the pure thermal effect on the cyclic phase transition behavior of the rod specimen, we have purposely removed the effect of the dislocation by training the specimen before the experiment. By synchronized measurements of the thermal and mechanical quantities, we systematically quantified the temperature, stress–strain and damping property of the material under displacement-controlled cyclic phase transition in the frequency range of 0.0004–1 Hz. We have identified the latent and hysteresis heats as the two heat sources and examined the effects of the heat release rate and the heat transfer rate on both the temperature variation and the cyclic stress–strain behavior of the rod. The experimental data are discussed and analysed and are further compared with the predictions from the lumped heat transfer modeling analysis for better understanding. Both experiment and analysis show that, among the many internal and external governing parameters, the deformation frequency, the heat source (latent and hysteresis heats), the heat transfer and the thermomechanical coupling have predominant effects in determining the temperature and the stress–strain response of the rod specimen. The key conclusions of the paper are as follows:

- The volume-averaged temperature as well as the stress–strain responses of the NiTi rod under cyclic phase transition of each frequency basically consist of two stages: the initial transient stage and the following steady-state stage. In the
transient stage, the competition between heat generation and heat transfer as well as the coupling between the thermal and mechanical quantities are reflected by the observed drifts of both the temperature variation and the stress–strain loop with the number of cycles. The drift in the stress–strain loop (drift up or down) follows the same trend as the temperature. The magnitude of the drift varies with the frequency in a non-monotonic way. In the steady-state, the temperature of the specimen oscillates around a mean temperature plateau and the stress–strain curves collapse onto a single loop. The achievement of the steady-state or the stabilized response is indicative of the balance between the heat generation and the heat transfer. It is the effect of heat (i.e. the resultant effect of the latent heat release/absorption, the hysteresis heat release and the heat transfer between the specimen and the ambient air) that leads to the stabilization of both the temperature and the stress–strain responses.

- Both experimental data and analysis show that the oscillation of the specimen’s temperature is mainly caused by the latent heat release/absorption during the forward/reverse phase transition while the difference between the steady-state mean temperature \( (T_{\text{mean}}) \) and the room temperature \( (T_{\text{mean}} - T_0 = D_s \Delta r f / \lambda) \) is mainly caused by the accumulation of the hysteresis heat in the cyclic phase transition. The steady-state temperature oscillation \( \Delta T_s \) increased with the frequency and eventually reached the saturated value of \( l_0 / \lambda \); while the steady-state mean temperature \( T_{\text{mean}}^s \) increased monotonically with the frequency due to the fast accumulation of the hysteresis heat. The variations of both \( \Delta T_s \) and \( T_{\text{mean}}^s \) with the frequency in the steady-state are determined by the ratio of the two time scales: the characteristic time \( t_p \) of the heat generation inside the material and the characteristic time \( t_h^o \) of heat transfer to the ambient. Theoretical predictions of both \( \Delta T_s \) and \( T_{\text{mean}}^s \) agree well with the experimental data without any fitting parameters.

- The stress in the cyclic loading strongly depends on the deformation frequency. The maximum stress (at the strain of 6%) in the stress–strain curve of the steady-state cycles increases monotonically with the frequency and can reach 611 MPa (400 MPa for the isothermal case) at the frequency of 1 Hz for the rod. The stress hysteresis loop area \( D_s \) (damping capacity of the material) is strongly coupled with the temperature variation in the cycle. \( D_s \) drifts with the number of cycles until it reaches its steady-state value \( D_s \). Both experiment and analysis show that \( D_s \) first increases then decreases with the frequency in a non-monotonic way. At the frequency of 1 Hz, \( D_s \) is about half of the isothermal value \( (D_0) \). These must be taken into account when using SMA as high-frequency damping devices.

- The significant loading frequency dependence of the thermal and mechanical responses of the NiTi SMA material reported in this paper strongly indicate that, in characterizing and modeling the stress–strain relationship of the SMA materials, one must clearly specify the time scale of loading and the time scale of the heat transfer between the specimen and the ambient. In addition to the factors such as the specimen geometry, the mean strain, the amplitude of the strain variation and the ambient condition, etc., the hysteresis loop area (hysteresis heat) and deformation frequency play important roles in the material’s thermomechanical responses and degradation through heat accumulation and heat transfer. More specifically, for a given ambient and specimen geometry, it is expected that the smaller the hysteresis of the material and the lower the frequency, the lower the temperature and the stress level and therefore the lower the material degradation will be.

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Appendix A. Governing equation and solution for the volume-averaged temperature \( T(t) \)

The method to build the governing equation for the volume-averaged temperature \( T(t) \) of the rod is basically from the heat transfer equation and lumped analysis (Cotta and Mikhailov, 1997). It is well known that, for a body of volume \( V \) and

![Fig. A1. Schematic diagram of the lumped analysis method for the transient heat transfer problem, from \( T(x, t) \) to the lumped temperature \( T(t) \).](image-url)
boundary surface $S$ (Fig. A1), the differential heat transfer equation can be derived from the local energy balance at any position $\mathbf{x}$, as
\[
\frac{dT(\mathbf{x}, t)}{dt} = \nabla \cdot k \nabla T(\mathbf{x}, t) + g(\mathbf{x}, t),
\] (A1)
where $\lambda = \rho c$ is the heat capacity per unit volume and $k$ is the heat conductivity, $g(\mathbf{x}, t)$ is the heat source. Lumped system analysis can be applied to Eq. (A1) for the body under the condition of low temperature gradient and can yield a very simplified expression of the volume-averaged temperature (for more details, please refer to Cotta and Mikhailov, 1997). An overall volume averaging of Eq. (A1) in region $V$ not only defines the volume-averaged temperature $T(t) = (1/V) \int_V T(\mathbf{x}, t) dV$ which is a function of time only, but also turns Eq. (A1) as
\[
\frac{dV}{dt} T(t) = \frac{1}{V} \int_V \nabla \cdot k \nabla T(\mathbf{x}, t) dV + g(t), \quad \mathbf{x} \in V, t > 0,
\] (A2)
where
\[
g(t) = \frac{1}{V} \int_V g(\mathbf{x}, t) dV. \tag{A3}
\]
We consider a NiTi SMA rod of radius $R$ and gauge length $L_0$ under displacement-controlled sinusoidal cyclic deformation of frequency $f$ (period $t_0 = 1/f$). The sinusoidal displacement $u(t)$, strain $\varepsilon(t)$ and strain rate $\dot{\varepsilon}(t)$ are respectively:
\[
u(t) = \frac{u_0}{2} (1 - \cos \omega t),
\]
\[
\varepsilon(t) = \frac{u_0}{2L_0} (1 - \cos \omega t),
\]
\[
\dot{\varepsilon}(t) = \frac{u_0 \omega}{2L_0} \sin \omega t,
\]
where $u_0$ is the amplitude of the displacement and $\omega$ is the angular frequency ($\omega = 2\pi f$). Now we derive the evolution of the specimen’s volume-averaged temperature $T(t)$ by lumped system analysis. To simplify the case, we used the following assumptions:

(1) Latent heat release/absorption and hysteresis heat release are the two internal heat sources of the material and their rates are approximated as being proportional to the applied strain rate and the square of the strain rate, respectively, without separating the loading-unloading process into different subsections (He and Sun, 2010a);
(2) The heat flow through the two grips are modeled as heat conduction through the two end cross-sections of the gauge (of length $L_0$) which experiences cyclic phase transition as shown in Fig. A2.

The volume-averaged heat source $g(t) = \int_V g(\mathbf{x}, t) dV$ in Eq. (A2) includes the latent heat (the total amount per unit volume is $l_0$) and the hysteresis heat ($D$ per unit volume). The latent heat comes from the phase transition and therefore the rate of latent heat release can be assumed to be proportional to the phase transformation rate or strain rate (i.e. positive in loading and negative in unloading). By assumption (1), we can write the latent heat release rate as $(l_0 \omega / 2) \sin (\omega t)$ which satisfies the condition that the integral of the latent heat release rate in loading and unloading is $l_0$ and $-l_0$ respectively (i.e. $\int_0^{t_0} (l_0 \omega / 2) \sin (\omega t) dt = -\int_{-t_0/2}^{t_0/2} (l_0 \omega / 2) \sin (\omega t) dt = l_0$). At the same time, the friction-type hysteresis heat is released and its rate increases with the strain rate and is always positive in both loading and unloading. Therefore we simply assume the

---

**Fig. A2.** (a) Heat transfer conditions in the real experimental and the model, and (b) the experimental setup for electric resistance heating.
hysteresis heat release rate is proportional to the square of the strain rate as \((D\omega/\pi)\sin^2(\omega t)\). The assumption we made is mainly to simplify the model and to make sure that hysteresis heat release is always positive during loading and unloading. We also assure the total hysteresis heat released in one cycle is \(D\), i.e. \(\int_0^{2\pi} (D\omega/\pi)\sin^2(\omega t) dt = \int_0^{\pi} (D\omega/\pi)\sin^2(\omega t) dt + \int_{\pi}^{2\pi} (D\omega/\pi)\sin^2(\omega t) dt = (D/2) + (D/2) = D\) (the hysteresis loop area). Now, we have

\[
g_{av}(t) = \frac{1}{V} \int_V g(x, t) dv = \frac{b_0}{2} \sin(\omega t) + \frac{D\omega}{\pi} \sin^2(\omega t) 
\]

We apply the divergence theorem to transform the volume integral of Eq. (A2) to a surface integral:

\[
\frac{1}{V} \int_V \nabla \cdot \mathbf{k} \nu T dv = \frac{1}{V} \int_S \nabla \cdot \mathbf{n} ds
\]

The surface integral \(\int_S k(\partial T/\partial n) ds\) includes the heat convection through the rod’s side surface with the heat flow \(Q_{conv} = -hA_{side}(T - T_0) = -h2\pi R_0(T - T_0)\) \((h)\) is the natural convection coefficient of the side surface in air and depends on the temperature difference (Holman, 2009)) and the heat conduction through the two end cross-sections with the heat flow \(Q_{cond} = -2k\pi R_0 dt/dx = -2k\pi R_0(\alpha(T - T_0))/L_0\), \(k\) is the conductivity and \(\alpha\) is a constant. Now \(\int_S k(\partial T/\partial n) ds\) (the total heat flow through the outer surface \(S\) of Eq. (A8) becomes:

\[
\int_S k(\partial T/\partial n) ds = -2\pi k\pi R^2 (\frac{T - T_0}{L_0}) - h2\pi R_0(T - T_0) = -\pi h2\pi R_0(T - T_0) 
\]

In the above, \(\bar{h}\) is the lumped effective heat convection coefficient of the side surface (i.e. already including the conduction contribution at the two end cross-sections of the rod) and \(\bar{h} = ((2ak/L_0)^2 + (2h/R)(R/2))\). It is seen that \(\bar{h}\) decreases with the increase in \(L_0\) and \(T\rightarrow h\) when \(L_0 > \sqrt{(akR)/h}\). So, with Eqs. (A7) and (A9), (A2) becomes

\[
\frac{\partial T}{\partial t} = -2\pi \frac{\bar{h}}{R} (T - T_0) + \frac{b_0}{2} \sin(\omega t) + \frac{D\omega}{\pi} \sin^2(\omega t) 
\]

Since both \(\bar{h}\) and \(D\) depend on the temperature and the temperature history for each loading frequency, Eq. (A10) is a nonlinear ordinary differential equation (ODE). For the purpose of simplicity and as a first approximation, we simply take \(\bar{h} = \bar{h}h\) and \(D = Dh\), i.e., using the steady-state values of \(\bar{h}\) and \(D\) for each frequency of loading. Then, Eq. (A10) becomes ODE with constant coefficients:

\[
\frac{\partial T}{\partial t} = -2\pi \frac{\bar{h}}{R} (T - T_0) + \frac{b_0}{2} \sin(\omega t) + \frac{D\omega}{\pi} \sin^2(\omega t) 
\]

Solving Eq. (A11) with the initial condition \(T|_{t=0} = T_0\), we have

\[
T(t) = \left(T_0 + \frac{D\bar{h}b_0}{2\pi h} \omega \right) + \left[ \frac{b_0}{2\pi h} \omega^2 - \frac{2D\bar{h}b_0^3}{\pi h (1 + 4\bar{h}b_0^2 \omega^2)} \right] e^{-\sqrt{\frac{\bar{h}}{R}} t} + \left[ \frac{b_0}{2\pi h} \omega \sin \left( \omega t - \frac{1}{\sqrt{1 + 4\bar{h}b_0^2 \omega^2}} \right) \right] 
\]

where \(\bar{h} = \sqrt{R^2/2D}\) is the characteristic time scale of the effective convection through the side surface. The first term on the right side of Eq. (A12) gives the steady-state mean temperature of the specimen, which is caused by the accumulation of the steady-state hysteresis heat (stress-strain loop area) of the phase transition. The second term is the specimen’s average temperature variation in the transient stage. It decays exponentially with time \(t\). Its sign depends on the values of \(b_0, D, \omega, \bar{h}\). The last term represents the temperature oscillations caused by latent heat and hysteresis heat, respectively. It is noticed that, different from latent heat release, there are two heat pulses (one in loading and one in unloading) that make the period of temperature change (due to the hysteresis heat) only half of that due to the latent heat. The key assumption underlying the lumped analysis is the small temperature gradient in the system, while the geometric shape of the system can be arbitrary (please see the book (Cotta and Mikhailov, 1997) on the lumped analysis method).

Remark: To check the reliability of the thermocouple measurement on the specimen’s average temperature, direct experimental observation on the temperature distribution at the specimen’s surface along the axis of the rod was performed by using a high time-resolution infrared camera (FLIR SC7700, at the capture rate of 5–200 frames/s) as shown in Fig. A3. Fig. A3(1) gives evidence how temperature field varied at high frequency, Fig. A3(3) gives evidence how temperature field varied at intermediate frequency, Fig. A3(5) and (6) show the difference between the mean temperature measured by the thermocouple and that by the infrared camera. During the phase nucleation and transformation, the temperature distribution is non-uniform in the beginning because of localized phase transformation, and then becomes more uniform due to rapid heat conduction within the specimen. For example, during the loading process in the first cycle of high frequency cyclic phase transition (see Fig. A3(1)), the nucleation and growth of martensite bands release latent heat and cause non-uniform temperature rise (see the non-uniform distributed temperature profiles in Fig. A3(2), line 6 and line 7 are the most significant ones). After 0.11 s (1/9 of the period), the temperature becomes uniform (from line 6 to line 8). The same phenomenon can be seen in the steady-state cycles during intermediate frequency (0.01 Hz) cyclic loading (see Fig. A3...
By comparing the specimen’s average temperatures from three thermocouples and infrared camera in Fig. A3(5) and (6), we can see there is no significant difference, so the specimen’s average temperature obtained from the thermocouple in the manuscript can characterize the volume-averaged temperature of the specimen.

References


